Employment Contracts for Earnings and Hours

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Abstract

Legal regulations of hours have become increasingly common, yet less is known about how workers' earnings and hours are set in employment contracts. In this paper, I document that workers commonly report changes in their hours, but the correlation between changes in earnings with changes in hours differs depending on whether their explicit contract pays them by the hour or not. To understand these patterns, I develop and estimate a frictional labor market model with implicit contracts motivated by risk-sharing and constrained by limited commitment. Although the model has no explicit hourly and non-hourly contracts, implicit contracts generate a larger correlation between changes in earnings and hours for workers with lower earnings, hours, and tenure as in the data. I find that hours variability plays a quantitatively important role to increase workers' output, which yields caution to policymakers when restricting hours variability. This is especially true for workers with high earnings and tenure who benefit the most from risk-sharing in implicit contracts.

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1 Introduction

Government policies in the United States increasingly regulate how workers' hours are set and adjusted at their jobs. Recently, the federal minimum salary threshold that exempts white-collar workers from overtime payments increased from \$683 to \$1,128 per week. Local governments, including New York City and San Francisco, have passed predictive scheduling laws that penalize firms for changing work schedules on short notice. Additionally, the states of Maine, Nevada, and Illinois now mandate that all workers receive paid time off from their employers. In order to effectively evaluate these policies, it is crucial to understand how workers' hours are determined in tandem with earnings.

While the textbook neoclassical model views hours as a choice made by workers who are paid what they produce in a competitive spot market, there is substantial evidence that firms play an important role in setting workers' hours (Altonji and Paxson [1992], Goldin [2014], Bick et al. [2022]). However, this literature on "hours constraints" typically views hours as a fixed feature of a job and has little to say about the variability of hours or earnings. The literature on implicit (or long-term) contracts provides an alternative framework to the spot market and hours constraints models, and previous research has found reduced form empirical evidence supporting this framework (Beaudry and DiNardo [1995], Ham and Reilly [2002]). Nevertheless, to the best of my knowledge no paper has developed and estimated a structural model with hours determined within implicit employment contracts to quantitatively assess how contracts influence workers' outcomes and evaluate the implications for government labor market policies.

In this paper, I make four contributions. First, I document in the Survey of Income and Program Participation that both hourly and non-hourly workers commonly report changes in their hours at their jobs, but changes in hours are associated with larger changes in earnings for hourly workers. Second, I develop a frictional labor market model where earnings and hours are determined as a component of an implicit employment contract motivated by risk-sharing and constrained by two-sided limited commitment. In the model, idiosyncratic shocks to firms' labor productivity generate a motive to vary hours at a job, and the implicit contract determines the pass-through of these shocks to earnings and hours. Third, I estimate the model to match the empirical dynamics of earnings, hours, and job mobility. Although the model does not have explicit hourly and non-hourly contracts, its implicit contracts generate a larger elasticity of earnings with respect to hours for workers with lower earnings, hours, and tenure as in the data. Thus, I argue that the distinction between *explicit* hourly and non-hourly contracts partially represents a distinction in the degree of risk-sharing in *implicit* contracts. Finally, I use the estimated model to evaluate the pass-through of shocks to workers' outcomes and the implications for restrictions on hours variability.

To begin, I use the Survey of Income and Program Participation (SIPP) to document that both hourly and non-hourly workers commonly report changes in their hours across surveys even without changing employers. In the SIPP, households are surveyed once each four months, and across surveys the standard deviation of the change in hours per week is 5.9 for hourly workers and 6.4 for non-hourly workers who do not switch firms. These large variations in hours are not driven by a small set of outliers, as about one-fourth of hourly and non-hourly workers report a change in hours across surveys that is greater than or equal to five in absolute value. One concern may be that these changes are largely measurement error (Bound et al. [2001]). However, I show that changes in hours are persistent across four surveys for both hourly and non-hourly workers, which means that they cannot be entirely explained by classical measurement error. Recent work using firms' administrative records has documented that changes in hours are common for hourly workers (Bound et al. [1994], Lachowska et al. [2023], Ganong et al. [2024]). Here, I additionally show that non-hourly workers report hours changes of similar magnitude to hourly workers in survey data that is persistent.

Next, I show that although changes in hours are common within a job for both hourly and non-hourly workers, hourly workers have more variable earnings when their hours change. Across surveys, a one percent increase in hours is associated with a 0.56% increase in earnings for hourly workers but only a 0.27% increase in earnings for non-hourly workers. Measurement error in hours will lead to a downward bias in these estimates, and temporary changes in hours may have a differential effect on earnings compared to persistent changes. To account for this, I use information about changes in hours across four consecutive surveys to disentangle the relationship between persistent changes in hours from temporary changes such as classical measurement error. For persistent changes, hourly workers still have large changes in earnings. A persistent one percent increase in hours is associated with a one percent increase in earnings for hourly workers but only a 0.50% increase in earnings for non-hourly workers. Previous statistical models of earnings and hours dynamics in survey data have found that on average changes in hours have a one-to-one association with changes in earnings (Abowd and Card [1987], Abowd and Card [1989], Altonji et al. [2013]), but I show that this relationship is heterogeneous across workers depending on whether they are paid by the hour or not.

In order to understand why different workers receive different employment contracts from firms, I develop a frictional labor market model with on-the-job search where risk-neutral firms and risk-averse workers cooperatively negotiate implicit contracts for earnings and hours. Within a match, changes in the firm's labor productivity and the worker's leisure preferences generate a motive to adjust hours within a job. The implicit contract agreed upon by the firm and worker specifies how shocks transmit to the worker's earnings and hours while also responding to outside offers that the worker receives via on-the-job search. Relative to recent papers studying how implicit contracts motivated by risk-sharing influence the transmission of labor productivity shocks to earnings or hourly wages (Rudanko [2011], Balke and Lamadon [2022], Souchier [2022]), this model additionally incorporates an hours margin of adjustment. Previous papers have studied hours within an implicit contract and found reduced-form empirical support for the predictions of a contracting model (Abowd and Card [1987], Beaudry and DiNardo [1995], Ham and Reilly [2002]). I augment these studies by introducing leisure preference shocks and search frictions.

Using the methods of Marcet and Marimon [2019], I derive the optimal history-contingent contract decisions for earnings and hours that reveal how a firm and worker optimally share risk. Relative to a model where workers set their own hours, the implicit contract *amplifies* the responsiveness of hours to productivity and preference shocks. The worker is willing to accept this amplification of hours variability because earnings are *insured* against the resulting output variation and are less variable with implicit contracts. However, if firms and workers lack commitment and can walk away from the contract as in Thomas and Worrall [1988], the extent to which the contract can amplify hours variations and insure the worker's earnings will be reduced. When commitment constraints rarely bind, the optimal contract approximates one with full commitment where the worker receives smooth earnings payments and hours vary in response to changes in labor productivity or leisure preferences. If they frequently bind, then the worker's earnings will vary in tandem with hours in order to satisfy the commitment constraints. This generates heterogeneity in the elasticity of earnings with respect to hours within a job based on the extent to which limited commitment constrains risk-sharing. While previous literature has considered differing job tasks as a reason for the distinction between hourly and non-hourly workers (Fama [1991], Haber and Goldfarb [1995], Hamermesh [2002]), different contracts arise in this model as a result of different levels of risk-sharing. Because hourly workers' earnings are more correlated with their hours than non-hourly workers, this suggests that they receive less insurance from firms in implicit employment contracts.

Using the simulated method of moments, I estimate the model to match the dynamics of earnings, hours, and job mobility observed in the SIPP. The labor productivity shock process and leisure preference shock process are separately identified based on the variability of earnings and hours. Although both shocks generate a motive to vary hours, labor productivity shocks will have a larger impact on workers' earnings. The estimated model is able to match well both the cross-section distribution of earnings and hours, the dynamics of earnings and hours, and the relationship between changes in earnings and changes in hours. To validate the model, I also show that the model generates more variable earnings for workers with lower earnings, hours, and tenure at their jobs. In the data, this is because these workers are more likely to be paid by the hour. In the model, it is because the outside option of non-employment is relatively more attractive for these workers, and this increases the likelihood that commitment constraints bind and generate earnings variability. The ability of the model to match these facts provides some evidence that the distinction between hourly and non-hourly workers partially reflects different levels of risk-sharing between firms and workers.

In the estimated model, I find that 49% of the variance of hours is generated by job-specific labor productivity shocks, and 13% is generated by worker-specific leisure preference shocks. This suggests that labor productivity shocks are the main driver of changes in hours within a job rather than workers' leisure preferences. Most of the remaining variance in hours is accounted for by fixed heterogeneity in worker types. Additionally, hours play an important role in the contract to amplify the effect of labor productivity shocks on match output. If hours remained fixed, then a one percent increase in match productivity would be associated with a one percent higher match output. However, workers with one percent higher match productivity have 1.4% higher output as a result of working 0.4% more hours on average. Furthermore, I find that workers' earnings are well insured against changes in output resulting from changes in their leisure preference and labor productivity shocks. Workers with one percent higher labor productivity shock have 1.4% higher output but only 0.154% higher earnings, implying that firms absorb about 89% of differences in output associated with changes in productivity. Similarly, workers with one percent higher leisure preference shocks have 0.4% lower output but only 0.17% lower earnings, implying that firms absorb 60% of output differences associated with changes in leisure preferences.

In the final part of the paper, I use the model to consider the implications of implicit contracts for government policies. I consider how workers' outcomes change when hours are exogenously set as a function of workers preferences and cannot adjust to changes in labor productivity shocks. When hours cannot adjust to changes in labor productivity shocks, output per worker declines in steady state by 12%, and workers are worse off by a consumption equivalence of 5% on average. Although the negative impact of restricting hours in an optimal contract is expected, these results emphasize that the variability of hours play an important role in the implicit contract to increase workers total expected output and earnings. It also provides caution to policymakers considering constraints on the variability of hours such as overtime premium, maximum hours of work, and predictive scheduling laws. However, these policies may also have benefits such as combatting monopsony power or work sharing that are not considered in this model, and the costs of restricting hours from the perspective of implicit contracts must be weighed against these benefits. Currently, policies regulating hours typically target specific groups of workers for whom the benefits of implicit contracting are likely to be smaller. For example, managers, professionals, and high income workers are excluded from overtime requirements, and workers in these occupations tend to have longer tenure on their jobs. Similarly, predictive scheduling laws target industries including retail and hospitality where tenure is low and turnover is high. When workers have longer tenure, the benefits from risk-sharing in implicit contracts are likely to be larger, and this can rationalize exempting these workers from certain labor market restrictions. Future policy discussions should continue to consider workers earnings and tenure when designing regulations of working hours.

Related Literature Starting with Baily [1974] and Azariadis [1975], many papers consider how risk-neutral firms provide insurance to their risk-averse workers against changes in productivity in an implicit (or long-term) contract. The goal of these original papers was to understand the sizeable fluctuations in employment over the aggregate business cycle despite limited changes in wage compensation. Building on this work, Harris and Holmstrom [1982] analyze wage growth when workers cannot commit to the contract, and Thomas and Worrall [1988] consider how wages fluctuate when neither firms nor workers can commit. Several papers have extended these frameworks to allow for an hours margin of adjustment (Sigouin [2004], Thomas and Worrall [2007]), and reduced form evidence supports predictions of the contracting model (Abowd and Card [1987], Beaudry and DiNardo [1991], Beaudry and DiNardo [1995], Ham and Reilly [2002] Lagakos and Ordonez [2011], Bellou and Kaymak [2012]). I build on this work by considering leisure preference shocks in addition to labor productivity shocks as a source of risk faced by workers and study heterogeneity in the amount of insurance firms provide to different workers. Additionally, I argue that implicit employment contracts can explain the distinction between explicit hourly and non-hourly contracts.

This paper also relates to the literature on the determinants of labor supply emphasizing the role of firms. Several papers in this literature consider or argue that hours are a rigid feature of a job, and workers who wish to change their hours must seek employment at a new firm (Kahn and Lang [1992], Altonji and Paxson [1992], Dickens and Lundberg [1993], Bloemen [2008], Chetty et al. [2011]). However, I highlight in the SIPP that reported changes in hours happen within jobs across a wide range of worker characteristics, and that these changes in hours are persistent. Furthermore, several recent papers have studied the empirical relationship between earnings and hours (Yurdagul [2017], Bick et al. [2022], Labanca and Pozzoli [2022], Shao et al. [2023]). They find that there is a weak or negative relationship between earnings and hours, particularly at longer hours of work, and view this as a result of the underlying returns to hours in production. My work emphasizes the dynamic relationship between changes in earnings and hours, and considers how risk-sharing between firms and workers can influence the relationship between earnings and hours observed in the data.

Moreover, this paper contributes to the literature studying frictional labor market models. A large part of this literature building on the work of Postel-Vinay and Robin [2002], Dey and Flinn [2005], Cahuc et al. [2006] consider both workers and firms to be risk-neutral, while my framework incorporates risk-averse workers and implicit contracts. While several recent papers have considered implicit contracts in a frictional labor market (Rudanko [2011], Balke and Lamadon [2022], Souchier [2022]), they abstract from an hours margin of adjustment. An exception is Sigouin [2004] who considers hours and aggregate productivity shocks in a frictional labor market; however, he does not consider leisure preference shocks or on-the-job search. My work is also related to Postel-Vinay and Turon [2010] and Lise et al. [2016], which use empirical dynamics of earnings to understand latent dynamics of match productivity in equilibrium search models. I additionally incorporate the dynamics of hours within jobs and leisure preference shocks influencing these hours.

Outline The rest of this paper is organized as follows. Section 2 provides an overview of the data facts regarding earnings, hours, and employment contracts. Section 3 presents the structural model of implicit contracts. Section 4 details the estimation, Section 5 discusses the analysis of the estimated model and policy implications. Section 6 concludes.

2 Empirical Dynamics of Earnings and Hours

This section documents three key data patterns regarding the dynamics of earnings and hours: (1) workers commonly report changes in their hours within a job and these changes are persistent, (2) the elasticity of earnings with respect to hours differs based on whether a worker is paid by the hour or not, and (3) hourly workers on average have shorter tenure, lower earnings, and shorter hours than observably similar non-hourly workers.

The main data source is the Survey of Income and Program Participation (SIPP) for panels starting in 1996, 2001, 2004, and 2008. In these surveys, households are interviewed once each four months for 32 to 48 months.[1] Studying hours for both hourly and non-hourly workers necessitates the use of survey data because administrative records either contain no information about hours or accurate information only for workers who are paid by the hour.[2] While self-reported hours in survey data contain measurement error (Bound et al. [2001]), I discuss how repeated observations in the SIPP can help to account for classical measurement error that is serially uncorrelated.

During each survey, individuals can report information about two jobs. For each job, individuals report their earnings in each month separately and usual hours of work per week. The specific question regarding hours is "How many hours per week did you usually work at all job-related activities at this job?" and is asked only once per survey. Although not specifically stated, I assume that this question captures the average hours of work per week at the job during the four-month survey. Because the question regarding hours is asked only once per survey rather than monthly, I sum the monthly earnings to survey-level job-specific earnings and then divide by the total number of weeks in the survey to arrive at a measure of job-specific earnings per week. In addition to job-specific earnings and usual hours, individuals are asked the number of weeks worked during the survey wave. I multiply weeks worked by job-specific usual hours of work per week and divide by the total number of weeks in the survey to arrive at a measure of job-specific hours per week. Thus, hours per week at a job may vary as a result of changes in usual hours of work per week or the fraction of weeks worked in the survey. At each job, individuals are also asked if they are paid by the hour or not, which I use to define hourly and non-hourly workers.[3]

To avoid dynamics in earnings and hours generated by education and retirement that are beyond the scope of this paper, I restrict the sample to individuals between the ages of 25 and 54. Because the SIPP over samples low-income households, I use longitudinal weights

[1] I use data from the 1996 to 2008 panels because they provide consistent variable definitions and timing. Starting in the 2014 panel, the SIPP transitioned to an annual frequency instead of a four-month survey frequency. Prior to 1996, research has found that the job identifiers in the SIPP contained substantial errors (Stinson [2003]). The 1990 to 1993 panels do have revised job identifiers based on tax records, and I plan to incorporate those in future work.

[2] Relative to other surveys that interview households annually or biennially such as the National Longitudinal Survey of Youth and Panel Survey of Income Dynamics, the SIPP between 1996 and 2013 allows for accurate measurement of employment dynamics at a higher frequency. The Current Population Survey interviews households each month and asks about hours of work in every survey, but questions on labor earnings are only asked in the outgoing rotation surveys and March ASEC supplements that happen one year apart. Across outgoing rotation surveys and Annual Social and Economic supplements, it is also not possible to identify for most workers if their employer changed. The CPS primarily serves as an employment survey, and the lack of detailed information on income motivated the creation of the SIPP as its name suggests.

[3] Unfortunately, more detailed questions about how workers are paid is not included. However, the PSID includes a more detailed set of options from which workers can report their payment scheme. In Appendix A.2, I show that the vast majority of workers not paid by the hour report that they are paid fixed salaries.

to make the sample representative of the U.S. population. In order to study the dynamics of earnings and hours, I include only observations for an individual at survey time t if that individual is also interviewed in the next three surveys $t+1$, $t+2$, and $t+3$. Thus, for each observation in my sample there are four consecutive surveys. To study dynamics within a job, I also require that their main employer remains the same across these four surveys, that the job starts before the first survey t, and that it ends after the fourth survey $t+3$. Furthermore, I drop individuals who report having a second job in any of the surveys. For individuals who the SIPP is not able to survey, earnings and hours will be imputed either with the values from an observably similar worker (hot-deck imputation) or with a value selected by the data editor (cold-deck imputation). These imputations will generate spurious dynamics in earnings and hours, so I drop all observations with imputed earnings or hours. Additionally, monthly earnings values in the SIPP are top-coded at $12,500 (implying a yearly salary of $150,000) to protect confidentiality, so I omit observations above this value.[4] Finally, starting in 2004, the SIPP allowed individuals to report "hours vary" as an answer to the usual hours of work question. I drop these observations because their usual hours per week is not known.

Appendix Table 13 shows the steps of the sample construction and observations remaining for hourly and non-hourly workers. A similar fraction of observations is dropped at each step except for two steps. First, the requirement that workers have the same employer in four consecutive surveys drops a larger fraction of hourly workers because they are more likely to change employers. Second, about 4% of non-hourly worker observations have top-coded earnings in at least one of the four surveys, while less than 1% of non-hourly worker observations have top-coded earnings. Summary statistics for the sample of hourly and non-hourly workers are shown in Table 1. In the sample, there are a similar number of hourly and non-hourly worker observations. Non-hourly workers have higher earnings and longer hours on average, while non-hourly workers are more likely to be female and have shorter tenure.

2.1 Workers commonly report changes in hours within a job.

How common are changes in hours within a job? The sample constructed contains workers i at survey t who do not switch firms in surveys $t+1$, $t+2$, and $t+3$. Table 2 shows how their hours of work per week change from surveys $t+1$ to $t+2$. Even for these workers who do not switch firms, they commonly report changes in their hours of work per week, and

[4] Top-coded earnings are replaced with the average monthly earning of observably similar workers who are also top-coded.

Table 1: Summary Statistics

	(1) Hourly	(2) Non-Hourly
Earnings per Week	557.74	933.99
	(317.14)	(476.78)
Usual Hours per Week	39.00	43.05
	(8.26)	(8.56)
Fraction of Weeks Employed	0.99	0.99
	(0.07)	(0.05)
Hours per Week	38.65	42.82
	(8.64)	(8.77)
Tenure (Years)	8.08	8.98
	(7.34)	(7.53)
Age	39.41	39.89
	(8.01)	(7.80)
Male	0.51	0.53
	(0.50)	(0.50)
Year	2003.36	2003.91
	(4.75)	(4.69)
Observations	88,591	85,392

Notes: This table shows means and standard deviations in parentheses for the key variables in the sample of SIPP workers. Observations are weighted using longitudinal weights. "Hours per week" is usual hours of work per week multiplied by fraction of weeks employed.

this is true for both hourly and non-hourly workers. The standard deviation of the change in hours per week is 5.9 for hourly workers and 6.5 for non-hourly workers. This variation is also not driven by a handful of outliers. Only about 60% of hourly and non-hourly workers report no change in their hours of work per week. Furthermore, 21% of hourly workers and 30% of non-hourly workers report a change in their hours of work per week of at least five in absolute value. Lastly, these changes in hours are not driven by particular subgroups of workers. In Appendix A.4, I show that workers commonly report changes in their hours per week even when not switching firms regardless of their age, education, sex, industry, or occupation.

Table 2: Changes in Hours within a Job

	(1) Hourly	(2) Non-Hourly
Mean ΔHours per Week	0.034	-0.033
	(5.883)	(6.445)
Fraction with ΔHours per Week $= 0$	0.645	0.628
	(0.479)	(0.483)
Fraction with $\lvert\Delta\text{Hours per Week}\rvert \geq 5$	0.218	0.296
	(0.413)	(0.457)
Fraction with $\lvert\Delta\text{Hours per Week}\rvert \geq 10$	0.097	0.141
	(0.296)	(0.348)
Observations	88,591	85,392

Notes: This table shows how hours vary within a job depending on workers' explicit employment contract. Standard deviations are in parentheses. The sample is restricted to workers who do not switch firms, and observations are weighted using longitudinal weights.

One concern may be that these changes are largely driven by measurement error, as validation studies have found substantial measurement error in survey-reported hours (Bound et al. [2001]). Additionally, these changes could largely be transitory rather than persistent. Using four repeated observations of the same individual at the same job, however, it is possible to test whether changes in hours are persistent. Formally, suppose reported hours h_{it} for individual i at time t consists of two components that are additively separable in logs:

$$\log h_{it} = p_{it} + m_{it} \tag{1}$$

Here, p_{it} is a persistent component of reported hours with $\text{cov}(p_{it}, p_{it+k}) > 0$. The transitory component m_{it} is serially uncorrelated and uncorrelated with the persistent component p_{it}. If there was classical measurement error in hours, then this would be captured by the transitory component m_{it}. However, m_{it} could also contain true temporary changes in hours. Across two surveys, it is not possible to know whether an observed change was caused by the persistent component p_{it} or the transitory component m_{it}.

However, using four consecutive surveys we can test whether there are any persistent changes. I will refer to the change in hours from periods t to $t+3$ as the outer change in hours, and the change in hours from periods $t+1$ to $t+2$ as the inner change in hours. Note that the covariance between the inner and outer change in hours is equal to the covariance of the inner and outer persistent change in hours:

$$\begin{aligned}
&\text{cov}(\log h_{it+3} - \log h_{it}, \log h_{it+2} - \log h_{it+1}) \\
&\qquad = \text{cov}(m_{it+3} - m_{it}, m_{it+2} - m_{it+1}) + \text{cov}(p_{it+3} - p_{it}, m_{it+2} - m_{it+1}) \\
&\qquad\quad + \text{cov}(m_{it+3} - m_{it}, p_{it+2} - p_{it+1}) + \text{cov}(p_{it+3} - p_{it}, p_{it+2} - p_{it+1}) \\
&\qquad = \text{cov}(p_{it+3} - p_{it}, p_{it+2} - p_{it+1}). \qquad (2)
\end{aligned}$$

This is because the measurement error is serially uncorrelated and uncorrelated with the persistent changes in hours, so the first three terms of the expand covariance are zero. Suppose that there are no persistent changes in hours, so $p_{it} = p_i$, so it is constant across time for each individual. Then the covariance of the inner and outer changes in hours would be zero by equation 2.

To test this hypothesis, we can regress the change in hours from $t+1$ to $t+2$ on the change in hours from t to $t+3$ for the sample of workers who do not switch firms in four consecutive surveys:

$$\underbrace{\log h_{it+2} - \log h_{it+1}}_{\Delta_{12}\log(\text{hours})} = \beta \underbrace{(\log h_{it+3} - \log h_{it})}_{\Delta_{03}\log(\text{hours})} + \gamma_1 X_{it} + \gamma_0 + \epsilon_{it}. \qquad (3)$$

Here, X_{it} are observable worker characteristics including survey time indicators, age indicators, 14 industry groups, 10 occupation groups, sex, race, and education. If there are in fact no persistent changes in hours and the covariance between these two variables is zero, then the resulting regression coefficient β would be zero. However, as shown in Table 3, the coefficient β is positive and statistically significant for both hourly and non-hourly workers. A one percent increase in hours from periods t to $t+3$ on average is associated with a 0.10% increase in hours from periods $t+1$ to $t+2$ for hourly workers and 0.08% change in hours for non-hourly workers. The persistence of changes in hours is slightly larger for hourly workers than for non-hourly workers, but the difference is not statistically significant. This means that reported changes in hours are persistent across time and cannot entirely be explained by transitory factors such as classical measurement error.

Table 3: Relationship between Inner and Outer Changes in Hours

	(1) Δ_{12}log(hours)	(2) Δ_{12}log(hours)
Δ_{03}log(hours)	0.104	0.080
	(0.008)	(0.009)
Sample	Hourly	Non-Hourly
Observations	88,591	85,392

Note: Standard errors in parentheses are clustered by individual. Sample includes individuals who do not switch firms in four consecutive surveys. Observations are weighted using longitudinal weights.

Previous work using firms' administrative records has documented that changes in hours within a job are common for hourly workers. This was originally noted in the validation study of the PSID, which examined the administrative records of a single large manufacturing company: "Company records for hourly workers also showed surprising variability in work hours and earnings from one pay period to the next" ([Bound et al., 1994, p. 347]). Additionally, Lachowska et al. [2023] analyze the hours reported in administrative records from Washington state and report: "(M)uch of the variation in hours appears to be within a job over time, as opposed to resulting from fixed employer and worker effects" (p. 17-18). Finally, Ganong et al. [2024] find in a payroll processing company's records that: "in one quarter of months earnings change by at least 21%... Virtually all of this earnings volatility is driven by fluctuations in hours." (p. 2).[5] Thus, it should not be surprising that hourly workers in survey data like the SIPP also report changes in their hours. In administrative records of hours, though, it is not possible to observe accurate information about the hours for workers who are not paid by the hour. A contribution of this paper is to document that non-hourly workers report changes in their hours in surveys of similar magnitude to hourly workers.

[5] This is more volatile than the changes in hours for hourly workers in the SIPP. For hourly workers in the SIPP, hours change by at least 16% in one quarter of surveys.

2.2 Earnings variability differs by contract type.

Both hourly and non-hourly workers commonly report changes in their hours within a job, but how variable are their earnings? Table 4 shows how the logarithm of earnings per week changes from surveys $t+1$ to $t+2$ for the sample of workers who do not switch firms. The standard deviation of the change in logarithm of earnings per week is 0.301 for hourly workers and 0.225 for non-hourly workers, meaning that hourly workers have more variable earnings. While only 8% and 11% of hourly and non-hourly workers report no change in the logarithm of their earnings per week, 27% of hourly workers and 38% of non-hourly workers report a change that is equal to the logarithm of 18 divided by 17. This is because the SIPP allows worker to report earnings in monthly or yearly amounts. In some surveys there are 18 weeks, while other surveys have 17 weeks, causing changes in earnings per week during the survey for workers who report monthly or yearly earnings to bunch here. The fact that more non-hourly workers have changes in earnings at this level suggests that they are more likely to report their earnings in monthly or yearly amounts. However, the difference in the variability of earnings is not entirely driven by this. About 38% of hourly workers report a change in their earnings that is greater than or equal to 10 log-points, while only 26% of non-hourly report similarly large changes in earnings.

How much of this variation in earnings is explained by changes in hours, and how do changes in hours relate to changes in earnings? To answer this question, I regress the change in the logarithm of workers' earnings per week e_{it} from periods $t+1$ to $t+2$ on the change in the logarithm of workers' hours per week from periods $t+1$ to $t+2$ for individuals who do not switch firms in four consecutive surveys t to $t+3$:

$$\underbrace{\log e_{it+2} - \log e_{it+1}}_{\Delta_{12} \log e_{it}} = \beta(\underbrace{\log h_{it+2} - \log h_{it+1}}_{\Delta_{12} \log h_{it}}) + \gamma_1 X_{it} + \gamma_0 + \epsilon_{it\cdot} + \epsilon_{t\cdot}. \tag{4}$$

The vector X_{it} contains observable worker characteristics including time indicators, age indicators, sex, 14 industry groups, 10 occupation groups, and race. Here, β is the elasticity of earnings with respect to hours, meaning that a one percent change in hours would be associated with on average a β percent change in earnings. However, this is not necessarily a causal elasticity. Instead, it just measures the statistical relationship between changes in earnings and hours. When estimated by ordinary least squares, the coefficient β measures this elasticity for a typical change in hours that may either be transitory or persistent. Results of estimating this regression by ordinary least squares are shown in the first two columns of Table 5. For hourly workers, a one percent increase in hours is associated with a 0.56% increase in earnings, while for non-hourly workers it is associated with only a 0.27%

Table 4: Changes in Earnings with a Job

	(1) Hourly	(2) Non-Hourly
Mean Δlog(Earnings per Week)	0.017	0.013
	(0.301)	(0.225)
Fraction with Δlog(Earnings per Week) = 0.0	0.083	0.111
	(0.277)	(0.315)
Fraction with $\lvert\Delta\log(\text{Earnings per Week})\rvert = \log\left(\frac{18}{17}\right)$	0.268	0.375
	(0.443)	(0.484)
Fraction with $\lvert\Delta\log(\text{Earnings per Week})\rvert \geq 0.10$	0.380	0.258
	(0.485)	(0.438)
Fraction with $\lvert\Delta\log(\text{Earnings per Week})\rvert \geq 0.20$	0.209	0.128
	(0.407)	(0.334)
Observations	88,591	85,392

Notes: This table shows how earnings change within a job by a workers explicit employment contract. Standard deviations are in parentheses. The sample is restricted to workers who do not switch firms, and observations are weighted using longitudinal weights.

increase. This estimate of the elasticity will measure an average of the relationship between persistent and transitory changes in hours and earnings. If transitory changes in hours are largely measurement error, then the estimated elasticity will be biased toward zero.

Table 5: Elasticity of Earnings with Respect to Hours

	(1)	(2)	(3)	(4)
	Δ_{12}log(earn)	Δ_{12}log(earn)	Δ_{12}log(earn)	Δ_{12}log(earn)
Δ_{12}log(hours)	0.557	0.271	1.024	0.504
	(0.019)	(0.020)	(0.068)	(0.093)
Sample	Hourly	Non-Hourly	Hourly	Non-Hourly
Estimation	OLS	OLS	IV	IV
First Stage F-Stat			148	82
R^2	0.194	0.063		
Observations	88,591	85,392	88,591	85,392

Note: Standard errors in parentheses are clustered by individual. Sample includes individuals who do not switch firms in four consecutive surveys. Observations are weighted using longitudinal weights.

To estimate the elasticity of earnings with respect to persistent changes in hours, we can use the outer change in hours $\log h_{it+3} - \log h_{it}$ as an instrument for the inner change in hours $\log h_{it+2} - \log h_{it+1}$. By the assumption that the transitory component m_{it} is uncorrelated with the persistent component p_{it}, the outer change in hours will be uncorrelated with the inner change in the transitory component of hours .

$$\mathrm{cov}(\log h_{it+3} - \log h_{it}, m_{it+2} - m_{it+1}) = 0, \tag{5}$$

Thus, the outer change satisfies the exogeneity condition for a valid instrument. Additionally, it will be correlated with the inner change in the persistent component of hours:

$$\mathrm{cov}(\log h_{it+3} - \log h_{it}, p_{it+2} - p_{it+1}) > 0. \tag{6}$$

As shown in Table 3, this is a relevant instrument with an F-statistic sufficiently large for valid statistical inference (Lee et al. [2022]). As shown in columns three and four of Table 5, the elasticity of earnings with respect to hours is larger for persistent changes in hours. Hourly workers with a one percent persistent increase in hours have on average a 1.024% increase in earnings, while non-hourly workers have on average a 0.504% increase in earnings. If transitory changes in hours are entirely classical measurement error and have no effect on earnings, then these estimates would suggest that 46% and 47% of the variance for the changes in hours are generated by measurement error for hourly and non-hourly workers, respectively. However, if transitory changes in hours are not entirely measurement error, then this calculation will over-state the importance of measurement error.[6]

Thus, whether measured by ordinary least squares or by the instrument, hourly workers' changes in earnings are more correlated with changes in their hours on their jobs. This elasticity is about twice as large in both cases. The elasticity for hourly workers is nearly one as would be expected for hourly workers below forty hours per week. Thus, although hourly and non-hourly workers experience changes in hours that are similar in magnitude, the relationship between changes in earnings and changes in hours is much stronger for hourly workers.

One concern is that changes in earnings and hours are mechanically more correlated for hourly workers by design of the survey. In the SIPP, earnings are measured at the monthly level. However, workers are allowed to report their earnings to the survey as paychecks, yearly, monthly, or weekly amount and have these converted to a monthly amount. Additionally, workers can choose to report an hourly wage and hours and have their monthly earnings calculated accordingly. Hourly workers presumably would be more likely to choose the hourly calculation, while non-hourly workers would likely choose a different way of reporting earnings. However, the reason the survey is designed this way is to reduce measurement error in earnings rather than having all respondents report a monthly amount. Thus, these measured elasticities likely provide relevant information about how changes in hours correlate with changes in earnings. As shown in Ganong et al. [2024], hourly workers do have month-to-month earnings variability that is highly correlated with hours changes, while non-hourly workers largely have stable earnings month-to-month. Another concern is that non-hourly workers are more likely to receive bonuses that hourly workers. Although changes in non-hourly workers' earnings have less of an effect on their earnings today, it may have a larger effect on their earnings when bonuses are paid at the end of the year. However, Grigsby et al. [2021] reported that bonuses account for at least 10% of earnings for less than 10% of workers. Thus, although the coefficients for non-hourly workers are likely to be downward biased as a result of bonuses, the fact that relatively few workers receive substantial bonuses suggests that this would not lead to a large change in the estimated results.

[6] Duncan and Hill [1985] report that the measurement error of annual changes in hours has a variance that is 80% of the variance of true annual changes in hours, which is substantially larger than the 46% and 47% calculated here. However, much of the measurement error found in validation studies of hours is mean reverting, while I assume the measurement error to be classical.

2.3 Differences between hourly and non-hourly workers

Previous work has considered the distinction between hourly and non-hourly workers to reflect the tasks done at their job (Fama [1991], Haber and Goldfarb [1995]). Manual and routine jobs will be paid by the hour because production is closely linked to hours, while cognitive and non-routine tasks will not be paid by the hour because hours are not easily observable. While there are substantial differences across occupations and education groups in the fraction of workers paid by the hour as shown in Appendix A.4, Hamermesh [2002] has shown that differences in job tasks and other observable worker characteristics can only provide a partial explanation between hourly and non-hourly jobs. He finds that in the last several decades of the 20th century, the fraction of workers paid by the hour was remarkably stable at around 55% of the labor force despite substantial changes in the observable characteristics of workers such as industry and occupation that would predict a sizeable decline in hourly contracts. I replicate and extend the results of Hamermesh [2002] in Appendix A.5 and show that the fraction of workers paid by the hour has continued to be stable despite further changes in observable worker characteristics.

In the next section, I develop a contracting model where differences in the elasticity of earnings with respect to hours differs across workers as a result of heterogeneous risk-sharing with firms. To derive some predictions for the model, here I compare the earnings, hours, and tenure of workers who are and aren't paid hourly in the following regression:

$$y_{it} = \beta \text{Hourly}_{it} + \gamma X_{it} + \epsilon_{it}. \tag{7}$$

The variable y_{it} is the outcome of interest, which is either the logarithm of earnings per week, the logarithm of hours per week, or years tenure at their job. The variable Hourly_{it} is an indicator equal to one if worker i is paid by the hours in survey t, and the vector X_{it} is a set of observable worker characteristics including education, age, sex, occupation, industry, indicators for the survey year, and indicators for the survey month. The coefficient β measures the average difference in the outcome variable for hourly workers relative to observably similar workers who are not paid by the hour. Results from the estimation of equation 7 are presented in Table 6. Even after controlling for a rich set of observable worker characteristics, hourly workers have on average about 0.288 lower logarithm of earnings per week, 0.108 lower logarithm of hours per week, and 0.8 fewer years of tenure at their job.

Table 6: Differences between Hourly and Non-hourly workers

	(1) log(earnings)	(2) log(hours)	(3) Tenure (Years)
Hourly	-0.288	-0.108	-0.800
	(0.007)	(0.004)	(0.096)
Controls	Yes	Yes	Yes
Observations	173,983	173,983	173,983

Note: Standard errors in parentheses are clustered by individual. Regression includes controls for age, sex, year, month, race, and education. Sample includes individuals who do not switch firms in four consecutive surveys. Observations are weighted using longitudinal weights.

3 Contracting Model

To understand how firms and workers agree to changes in earnings and hours at a job, this section develops a model where firms offer implicit contracts and compete for workers in a frictional labor market. The implicit contracting framework builds on Thomas and Worrall [1988] by adding an intensive hours margin of adjustment, while the frictional labor market builds on Dey and Flinn [2005] by allowing firms to deliver values via implicit contracts. There are three key ingredients of the model to match the three empirical facts. First, a worker's hours will vary at her job as a result of shocks to either her leisure preferences or her firm's labor productivity. Second, her earnings will not necessarily vary in tandem with her hours or production as the result of risk-sharing in an implicit contract with her employer. Finally, two-sided limited commitment will constrain risk-sharing in implicit contracts and generate dynamics in earnings and hours that differ across workers. In particular, earnings variability will be higher for workers who have lower earnings, shorter hours, and shorter tenure at their jobs.

3.1 Agents and Environment

The labor market is frictional and in a steady state equilibrium. On one side of the market, there is a unit mass of workers who are heterogeneous in their average labor productivity type ψ_i and average leisure preference type θ_i jointly distributed according to $F_{\psi\theta}$. On the other side of the market, there is a continuum of ex-ante identical firms who produce a homogenous good. Time t is discrete and runs forever. Both firms and workers live forever and discount the future with a common factor $\beta \in [0, 1)$. The probabilities that non-employed and employed workers contact a job at another firm are given by λ_n and λ_e, respectively.

3.2 Production and Preferences

Production is constant returns to scale across workers within a firm and across firms within the economy. Thus, a firm may consider the production of each of its workers independently of other workers' production. Output from a matched worker i and job j each period depends on the worker's productivity type ψ_i, a job-specific labor productivity shock x_{jt}, and the worker's hours h_{it}. The production function is:

$$y_i(h_{it}|x_{jt}) = \psi_i x_{jt} h_{it}^{\alpha}. \tag{8}$$

Labor productivity shocks x_{jt} follow a Markov process $f_x(x'|x)$, and the initial value of the labor productivity shock at the start of a job is drawn from a distribution F_0. These shocks are job-specific and uncorrelated with shocks at other firms and to other workers. The parameter $\alpha \in (0, 1]$ measures the elasticity of production with respect to hours. If there are diminishing returns to hours in production (i.e. $\alpha < 1$), output at a fixed labor productivity shock x_{jt} would be maximized by smoothing hours across periods. When the labor productivity shock varies over time, however, the total expected production from the worker can be increased by shifting hours from low productivity periods to high productivity periods.

The flow utility to the worker each period depends upon her consumption c_{it}, hours h_{it}, preference type θ_i, and worker-specific leisure preference shock ϕ_{it}. Preferences are additively separable in consumption and hours and are represented by the utility function:

$$u_i(c_{it}, h_{it}|\phi_{it}) = \frac{c_{it}^{1-\tau} - 1}{1-\tau} - \theta_i \phi_{it} \frac{h_{it}^{1+\gamma}}{1+\gamma}. \tag{9}$$

Leisure preference shocks follow a Markov process $f_\phi(\phi'|\phi)$. These shocks are specific to a worker and will follow her across firms and into non-employment. The parameter $\gamma > 0$ captures the increasing marginal disutility from additional hours of work at a fixed leisure preference level ϕ_{it}. Although the worker would prefer to smooth her hours at a fixed preference shock level ϕ_{it}, her total expected value from the job will increase if hours of work are shifted from high leisure preference shock periods to low leisure preference shock periods

The parameter $\tau > 0$ measures the worker's desire to smooth her consumption across time. I assume that the worker lacks access to any financial instruments such as savings and borrowing, so the worker will be hand-to-mouth and consume her labor earnings e_{it} when employed.[7] When non-employed, the worker has zero hours of work and receives consumption b from home production that is common to all workers. Risk-aversion plus a lack of financial markets means that the worker would accept a reduction in her total expected earnings for a sufficiently large reduction in the variability of her earnings. In contrast to the worker, firms are risk-neutral and desire only to maximize their total expected profits from the match with the worker. Risk-neutrality implies that the firm would accept an increase in profit variability within the match in exchange for an increase in total expected profits from the match. Thus, there is an opportunity for both firms and workers to benefit by sharing risk in an implicit contract.

3.3 Implicit Contracts and Limited Commitment

Consider a worker i starting a job j at a time normalized to zero. In what follows, I drop the i and j subscripts for readability. The worker's leisure preference shock ϕ_t and the job's labor productivity shock x_t are observed by both the firm and worker each period prior to the choice of earnings and hours. Additionally, if the worker successfully contacts a new firm by searching in the frictional labor market, the labor productivity shock that she draws at this new firm $\hat{x}_t$ is observed by both herself and the current firm prior to decisions. If the worker is unsuccessful in search, then $\hat{x}_t = \emptyset$ and her only outside option is non-employment. Let $\eta_t = \{\phi_t, x_t, \hat{x}_t\}$ be the current state variables, and let $\eta^t = \{\eta_0, ..., \eta_t\}$ be the history of shocks up through period t. Because the state of the world η_t is observed by both the firm and worker, it is possible to write an implicit contract $\mathcal{C}$ where labor earnings $e_t(\eta^t)$, hours of work $h_t(\eta^t)$, flow profits $\pi_t(\eta^t)$, and dissolution $d_t(\eta^t)$ are time-specific functions of the history of states η^t:

$$\mathcal{C} = \{e_t(\eta^t), h_t(\eta^t), \pi_t(\eta^t), d_t(\eta^t)\}_{t=0}^{\infty}. \tag{10}$$

For readability in what follows, I omit the functional argument for the decision variables.

Both the firm and worker can each unilaterally dissolve the contract at any time for an outside option. The firm's outside option is to fire the worker and receive a terminal value of zero from the job. Define $W_i^*(\phi, x)$ to be the highest value that a firm can offer a worker i with leisure preference shock ϕ and labor productivity shock x while the firm earns non-negative total expected profits. The worker's outside option each period depends on whether she contacts a new firm in the frictional labor market and the labor productivity shock $\hat{x}$ that she draws at this new firm. Define $\hat{W}(\phi, \hat{x})$ to be the value of the worker's best outside

[7] Adding savings to the model substantially complicates the solution. With incomplete asset markets, workers will still demand to receive some insurance from their firms against shocks in addition to self-insuring themselves against changes in their earnings. Recent work by Souchier [2022] finds that self-insurance through asset markets partially crowds out insurance provided by firms in implicit contracts.

option:

$$\hat{W}_i(\phi, \hat{x}) = \max\{N_i(\phi), W_i^*(\phi, \hat{x})\}. \tag{11}$$

Here, $N_i(\phi)$ is the value of non-employment to the worker with leisure preference shock ϕ, and $W^*(\phi, \hat{x})$ is the maximum value that the new firm would be willing to offer the worker. When the worker does not contact a new firm in search so $\hat{x} = \emptyset$, then the maximum value that she can receive from an outside option firm $W^*(\phi, \emptyset) = \emptyset$. Because the firm and worker can unilaterally dissolve the contract, it will be constrained by limited commitment and must be self-enforcing as in Thomas and Worrall [1988]. If either the firm or worker decides to dissolve the contract, they lose contact and cannot form a new match with each other in future periods.[8] The ability of the firm to freely dissolve the contract is consistent with at will employment laws in all of the United States except Montana. For the worker, the freedom to dissolve the contract for an outside option firm is consistent with a lack of cumbersome non-compete clauses. When the match is dissolved by either party, the firm receives its outside option value of zero. In contrast, the worker either receives the non-employment value or bargains a new contract with her new firm. Define $\mathcal{B}^W(\phi, x, \hat{x})$ to be the value that the worker can bargain with her new firm if the match is dissolved as described in the next section.

3.4 Contract Negotiation and a Recursive Formulation

The firm and worker cooperatively bargain the implicit contract $\mathcal{C}$ in order to maximize the product of their surplus values (Nash [1953]). Define $W(\mathcal{C}, \eta_0)$ and $J(\mathcal{C}, \eta_0)$ to be the values that worker and firm receive from contract $\mathcal{C}$. The cooperatively bargained contract solves:

$$\max_{\mathcal{C}} \quad \left(W(\mathcal{C}, \eta_0) - \hat{W}_0\right) J(\mathcal{C}, \eta_0), \tag{12}$$

subject to

1. Budget constraint:

$$e_t + \pi_t = y(h_t | x_t). \tag{13}$$

2. Firm's commitment constraint:

$$(1 - d_t)\Big(\pi_t + \mathbf{E}\Big[\sum_{k=1}^{\infty} \beta^k (1 - D_{t+k})(1 - d_{t+k})\pi_{t+k} \;\Big|\; \eta^t\Big]\Big) \geq 0. \tag{14}$$

[8] However, Fujita and Moscarini [2017] show empirically that recall is in fact common for the unemployed in the SIPP, where 40% of unemployment spells end in a recall to the previous employer. Most of these recalls are not for workers who report being on temporary layoff. Incorporating this recall margin into an implicit risk-sharing framework between firms and workers could be an interesting avenue for future research.

3. Worker's commitment constraint:

$$(1-d_t)\Big(u(e_t,h_t|\phi_t)+\mathbf{E}\Big[\sum_{k=1}^{\infty}\beta^k\big\{(1-D_{t+k})(1-d_{t+k})u(e_{t+k},h_{t+k}|\phi_{t+k}) \\ +(1-D_{t+k})d_{t+k}\mathcal{B}^W_{t+k}\big\}\;\Big|\;\eta^t\Big]\Big)\geq(1-d_t)\hat{W}_t. \quad (15)$$

Here, $D_t = \max\{d_k\}_{k=0}^{t-1}$ tracks whether either the firm or worker dissolves the contract prior to period t. Earnings for the worker will certainly be positive because of the functional form assumption for utility, but the flow profits to the firm π_t could be negative as long as the total expected profits in future periods are sufficiently positive. This is captured by the firm's commitment constraint. Similarly, the worker must receive a total expected value at each point in time that is weakly greater than her outside option value $\hat{W}_t$.

Although the optimal contract consists of an infinitely large set of functions that each depends on a set of arguments that grows ever larger over time, it is possible to recursively solve for the optimal contract decisions using the ideas of Marcet and Marimon [2019]. As described in the following proposition, the optimal contract problem can be written as a recursive saddle point problem where decisions in period t depend not on the full history of states η^t but only the current state η_t and the previous periods earnings e_{t-1}.

Proposition 1. In any Pareto efficient contract, the following conditions are satisfied:

1. For all states of the world $\eta_t = (\phi_t, x_t, \hat{x}_t)$, there exists a lower bound on earnings $e_{\min}(\eta_t)$ and upper bound on earnings $e_{\max}(\eta_t)$.

2. The optimal decision in period t are:

   (a) The match dissolves if $e_{\max}(\eta_t) < e_{\min}(\eta_t)$:

$$d_t = \begin{cases} 1 & \text{if } e_{\max}(\eta_t) < e_{\min}(\eta_t), \\ 0 & \text{if } e_{\max}(\eta_t) \geq e_{\min}(\eta_t). \end{cases} \quad (16)$$

   (b) For all $t > 0$ when $d_t = 0$, earnings e_t are set according to:

$$e_t = \begin{cases} e_{\min}(\eta_t) & \text{if } e_{t-1} < e_{\min}(\eta_t), \\ e_{t-1} & \text{if } e_{\max}(\eta_t) \geq e_{t-1} \geq e_{\min}(\eta_t), \\ e_{\max}(\eta_t) & \text{if } e_{\max}(\eta_t) < e_{t-1}. \end{cases} \quad (17)$$

(c) For all $t \geq 0$ when $d_t = 0$, hours h_t are set according to:

$$h_t = \left(e_t^{-\tau} \alpha \frac{\psi_i}{\theta_i} \frac{x_t}{\phi_t} \right)^{\frac{1}{1-\alpha+\gamma}}. \tag{18}$$

Proof: Appendix

In this proposition, the lower bound on earnings $e_{\min}(\eta_t)$ represents the lowest earnings that the contract must promise the worker in order to provide her value $\hat{W}_t$. Similarly, the upper bound on earnings $e_{\max}(\eta_t)$ represents the highest earnings that the contract can provide the worker while the firm receives non-negative total expected profits. If the lower bound on earnings is greater than the upper bound on earnings for a given state η_t, the firm's and worker's commitment constraints cannot be satisfied simultaneously, so they must separate; this is captured in equation 16. Otherwise, the contract will attempt to pay the worker the same earnings as the prior period; this is because the worker is risk-averse while the firm is risk-neutral, so an optimal risk-sharing agreement provides smooth earnings payments to the worker. When a change in state variables causes a binding commitment constraint at the previous earnings level, however, the contract adjusts the worker's earnings by as little as possible to satisfy the constraint. This adjustment impacts the worker's earnings not only today but also in future periods so long as commitment constraints do not bind. Although earnings for the worker will be smoothed across time, hours will adjust in response to shocks to labor productivity x_t and leisure preference shocks ϕ_t. This is because correlating hours with productivity shocks increases the total expected output from the match, and correlating hours with leisure preference shocks increases the total expected value of the worker. When a commitment constraint binds, however, the contract optimally adjusts the worker's hours as well as her earnings to satisfy the constraint; this is captured by hours in period t depending upon earnings e_t in period t.

This proposition provides a recursive formulation of the values provided by the contract at all histories depending on earnings in the previous period and the current state of the world. We can define $J(e_{-1}, \phi, x, \hat{x})$ and $W(e, \phi, x, \hat{x})$ to be value functions for the firm and the worker based on the previous earnings $e_{\text{-}1}$ and current state of the world:

$$\begin{aligned} J(e_{-1}, \phi, x, \hat{x}) = (1-d)\Bigg(\pi + \beta \int_{x'} \int_{\phi'} (1-\lambda_e) J(e, \phi', x', \emptyset) \\ + \lambda_e \Big(\int_{\hat{x}'} J(e, \phi', x', \hat{x}') dF_0(\hat{x}') \Big) dF_x(x'|x) dF_\phi(\phi'|\phi) \Bigg), \end{aligned} \tag{19}$$

$$W(e_{-1},\phi,x,\hat{x}) = (1-d)\Bigg(u\left(e,h|\phi\right) + \beta\int_{x'}\int_{\phi'}(1-\lambda_e)W(e,\phi',x',\emptyset) + \lambda_e\Big(\int_{\hat{x}'}W(e,\phi',x',\hat{x}')dF_0(\hat{x}')\Big)dF_x(x'|x)dF_\phi(\phi'|\phi)\Bigg) + d\mathcal{B}^W(\phi,x,\hat{x}). \quad (20)$$

Here, decisions for dissolution d, earnings e, and hours h depend on the current leisure preference and labor productivity shocks $\{\phi, x, \hat{x}\}$ as well as the previous period's earnings e_{-1} as given in the Proposition 1.

Because the chosen contract maximizes the product of the firm and worker surplus, they will certainly agree to a contract that is Pareto efficient. This means that there are no feasible contracts that can make the firm better off without making the worker worse off and vice versa. The above proposition casts the Pareto frontier in terms of the worker's initial earnings e_0, which will be decided to maximize the product of the firm's and worker's surplus:

$$e_0(\phi,x,\hat{x}) = \arg\max_{e} \quad \Big(W(e,\phi,x,\hat{x}) - \hat{W}(\phi,\hat{x})\Big)\, J(e,\phi,x,\hat{x}) \quad (21)$$

Thus, solving for the optimal contract amounts to solving for the initial earnings level e_0 that maximizes the product of the surplus values received by the firm and worker. The value that the worker receives from unemployment with leisure preference shock ϕ is then given by:

$$N(\phi) = u(b,0|\phi) + \beta\int_{\phi'}\left\{\lambda_n\left(\int_{x'}W(e_0,\phi',x',\emptyset)dF_0(x')\right) + (1-\lambda_n)N(\phi')\right\}dF_\phi(\phi'|\phi) \quad (22)$$

Here, the earnings e_0 that the worker receives when finding a job next period depends on the labor productivity shock x' that she draws from F_0 as well her leisure preference shock ϕ' and is determined by the problem in equation 21.

Finally, the value that the worker receives when the match dissolves $\mathcal{B}^W$ is given by:

$$\mathcal{B}^W(\phi,x,\hat{x}) = \max\{N(\phi), \quad W(e_0(\phi,\hat{x},\emptyset),\hat{x},\emptyset), \quad W(e_0(\phi,\hat{x},x),\hat{x},x)\} \quad (23)$$

The first term in the maximization set represents the scenario when either the worker does not meet a new firm in the frictional labor market so $\hat{x} = \emptyset$ or the labor productivity shock at the new firm $\hat{x}$ is so low that the worker would prefer non-employment. The second term represents the scenario when the worker meets a new firm in the labor market that is superior to non-employment, but the labor productivity shock at her current firm x is so low that the worker would prefer non-employment. In this case, it is optimal for the worker to bargain against non-employment as her outside option. The final case represents the scenario where she meets a new firm and both the labor productivity shock at her current firm x and the labor productivity shock at the new firm $\hat{x}$ are sufficiently high that the worker would prefer each firm to non-employment. In this case, the worker will use her current firm's labor productivity shock x when bargaining with the new firm with labor productivity shock $\hat{x}$.

3.5 Characterizing Risk-Sharing and Limited Commitment

The worker's hours are optimally set each period such that the worker's marginal rate of substitution of hours for earnings is equal to the marginal rate of transformation of hours into production:

$$-\frac{u_h(e,h|\phi)}{u_e(e,h|\phi)} = y_h(h|x) \tag{24}$$

The marginal rate of transformation can also be thought of as the firm's marginal rate of substitution of hours for earnings where the firm's utility function is defined based on the budget constraint $\pi = y - e$. Note that the specific functional equations for utility and production are not necessary in order for an optimal implicit contract to exist. They are necessary, however, to easily characterize how hours respond to shocks in the equation.

To understand how risk-sharing and limited commitment influence the variation in earnings and hours, it is useful to consider two extreme cases: fully enforceable contracts and spot markets. With enforceable implicit contracts, the optimal contract does not need to adjust based on the firm's and worker's commitment constraints. It needs only to consider the worker's outside option value in the initial period $\hat{W}_0$ when setting earnings and hours as shown in the following proposition.

Proposition 2. If employment contracts were fully enforceable, then earnings are constant at all points in time in the match ($e_t(\eta^t) = e_0$), and hours are set as:

$$h_t = \left(e_0^{-\tau}\alpha\frac{\psi_i}{\theta_i}\frac{x_t}{\phi_t}\right)^{\frac{1}{1-\alpha+\gamma}}. \tag{25}$$

Proof: Appendix.

Fully enforceable contracts mean that the contract can ignore binding commitment constraints in future periods. Earnings are set in the initial period to maximize the product of the firm's and worker's surplus values and remain constant throughout the match because the worker is risk-averse. Hours each period optimally respond to shocks to labor productivity and leisure preferences. The elasticity of the worker's hours in response to shocks in fact does not depend at all upon her risk-averse level τ but only on the elasticity of production with respect to hours α and the inverse Frisch elasticity of labor supply γ.

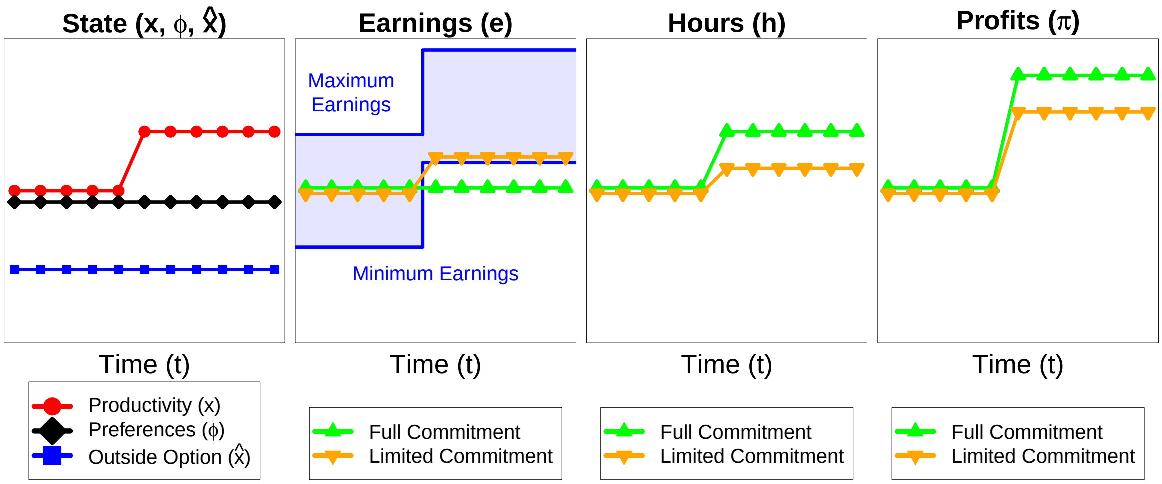
In the other extreme, suppose the employment contracts behave like spot market contracts where the worker chooses her own hours and was paid exactly what she produced. This would be the case if worker's outside option was always another firm with an identical labor productivity shock as her current firm. In this case, the worker would set her hours such that the marginal benefit of additional consumption from additional production was equal to marginal cost of additional hours so that:

$$h_t = \left(\alpha \frac{\psi_i^{1-\tau}}{\theta_i} \frac{x_t^{1-\tau}}{\phi_t}\right)^{\frac{1}{1-\alpha(1-\tau)+\gamma}}, \qquad \text{and} \qquad e_t = \psi_i x_t h_t^{\alpha}. \tag{26}$$

Here, we see that the elasticity of hours with respect to shocks is smaller when workers set their own hours and consume their production compared to the implicit contracting model because $\tau > 0$. Because hours variation will affect her consumption, the risk-averse worker attenuates the variability of her hours in response to shocks.

The variability of earnings and hours under implicit contracts constrained by limited commitment sits somewhere between these two extremes depending on the quality of the worker's outside option. If the worker has a better outside option, then it will be more likely that the firm's and worker's commitment constraints will bind and require a change in earnings to maintain the match. Figure 1 shows how the contract behaves in response to an increase in the job's labor productivity shock. If the contract was fully enforceable, then the contract increases the worker's hours without any increase in her earnings. Although the worker is worse off during the high productivity period because she receives no compensation for her additional hours, the firm benefits by taking the additional production as additional profits. The converse would happen in response to a negative labor productivity shock. Although either the firm or worker is worse of ex-post, they both benefit ex-ante from the risk-sharing agreement. However, if the worker had a better outside option, then she could threaten to separate from the firm because her hours are too high and earnings too low. Wanting to retain its worker, the firm increases the worker's earnings and reduces her hours to match her outside option value. Because the worker is risk-averse and has increasing marginal disutility of hours, the firm adjusts her earnings and hours not only during the period with high productivity but also in future periods to smooth these changes over time. These future promises allow the firm to optimally take advantage of the variation in productivity while providing value to the worker.

Figure 1: Impulse Response Functions



Note: This figure shows how earnings, hours, and profits respond in the model to an increase in labor productivity under full and limited commitment.

4 Estimation

In order to evaluate the pass-through of shocks to workers' outcomes and to quantify the importance of hours variability in the contract, I estimate the model to match the dynamics of earnings, hours, and job mobility in the SIPP.

4.1 Data Moments

I use data from SIPP panels starting in 1996, 2001, 2004, and 2008 and weight observations using longitudinal weights. There are two sets of moments that I target: job mobility and the dynamics of earnings and hours. For job mobility, I use the monthly labor force status variable at the time of the survey and construct trimesterly transition probabilities for starting a job (UE-rate), losing a job (EU-rate), and changing jobs (EE-rate).

For earnings and hours dynamics, I select the sample of interviews for workers whose earnings and hours are not top-coded or imputed for all four months in the survey. Then, I aggregate monthly earnings during the four-month wave and divide by the number of weeks in the wave to arrive at a measure of weekly earnings. For hours, I multiply usual hours of work per week by number of weeks worked during the wave and divide by the number of weeks in the wave to arrive at a measure of hours per week. Finally, I residualize hours per week and earnings per week by indicators for time, age, sex, race, education level, 10 industry groups, and 14 occupation groups.

4.2 Externally Set Parameters

The length of a period is assumed to be four months as in the SIPP. I set workers' relative risk-aversion in consumption τ to approach one, so they have logarithmic utility in consumption. This is on the lower end of estimates in the literature for risk-aversion (Layard et al. [2008]). However, because I do not allow workers to save, their desire for smooth earnings across periods will be lower than if workers were able to save. I set the inverse of the Frisch elasticity γ equal to 2, which is common in the literature on labor supply. The discount rate $\beta = 0.99$ for the four-month period, which implies an annual interest rate of 3.1%. Finally, the returns to hours in production α is set equal to 1.[9]

Table 7: Externally Set Parameters

	Value
Discrete Period Length	Four Months
Coefficient of Relative Risk-Aversion (τ)	$\rightarrow 1$
Inverse Frisch Elasticity (γ)	2.0
Discount Factor (β)	0.99
Returns to Hours in Production (α)	1.0

Note: This table shows the values of the parameters set outside of estimation as described in Section 4.2.

4.3 Parameterization and Moment Selection

The model allows for permanent differences in individuals' productivity ψ_i and preferences for leisure θ_i that are jointly distributed according to $F_{\psi\theta}$. Because the model must be solved separately for each worker type i, I assume there are only nine types of workers in the economy for computational reasons. The distribution $F_{\psi\theta}$ is assumed to be a nine-point

[9] The returns to hours in production is difficult to identify from data on earnings and hours alone. Pencavel [2015] using data on the production of World War II factory workers finds that production is roughly linear in hours below 50 hours of work per week and then flattens. Several papers on the "part-time wage penalty" require a return to hours in production substantially larger than one to match the data (Aaronson and French [2004], Erosa et al. [2022]), while complementarities in production would generate non-linear returns to hours in production (Yurdagul [2017], Bick et al. [2022], Shao et al. [2023]).

Gauss-Hermite approximation of a joint log-normal distribution.[10]

$$\begin{bmatrix} \log(\psi_i) \\ \log(\theta_i) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\psi \\ \mu_\theta \end{bmatrix}, \begin{bmatrix} \sigma_\psi^2 & \rho_{\psi\theta}\sigma_\psi\sigma_\theta \\ \rho_{\psi\theta}\sigma_\psi\sigma_\theta & \sigma_\theta^2 \end{bmatrix} \right) \tag{27}$$

This yields five parameters to be estimated underlying the distribution of worker types $(\mu_\psi, \mu_\theta, \sigma_\psi, \sigma_\theta, \rho_{\psi\theta})$. Because I use only nine-points, this will not be a good approximation of the joint normal distribution. However, it allows for permanent differences in worker types to influence the distribution of earnings and hours. In order to estimate these parameters, I target the first and second order moments from the cross-sectional distribution of earnings and hours.

The Markov processes for preferences f_ϕ and job-specific labor productivity f_x are assumed to follow a logarithmic autoregressive process of order one with a normally distributed stochastic term:

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t} \qquad \epsilon_{x,t} \sim \mathcal{N}(0, \sigma_x^2), \tag{28}$$

$$\phi_{t+1} = \rho_\phi \phi_t + \epsilon_{\phi,t} \qquad \epsilon_{\phi,t} \sim \mathcal{N}(0, \sigma_\phi^2). \tag{29}$$

Computationally, I approximate these distributions as a discrete Markov process with five states using the method proposed by Rouwenhorst [1995]. Relative to other methods, Rouwenhorst's method is more accurate in approximating an autoregressive process even when the persistence parameters are relatively high (Kopecky and Suen [2010]). In order to estimate the parameters $(\rho_x, \sigma_x, \rho_\phi, \sigma_\phi)$, I target the variability and persistence of changes in earnings and hours.

As shown in Proposition 1, when either productivity or preferences change, the optimal implicit contract will adjust the worker's hours in response. Thus, separately identifying the shock processes is challenging. However, I argue that productivity shocks should have a larger impact on the variability of worker's earnings than preference shocks.[11] This is because a preference shock impacts output only via a change in hours, while a productivity shock will impact output both through changes in hours and through the change in productivity. Be-

[10] This is the same as how Fan et al. [2024] model individual heterogeneity in ability to learn and tastes for leisure.

[11] A formal proof of identification could proceed similarly to the proof in Balke and Lamadon [2022]. In their model of earnings dynamics in implicit contracts, workers experience changes in labor productivity that affect only their own output, and firms experience changes in labor productivity that affect the output of all workers at the firm. They show that five periods of data on a worker's earnings and her co-worker's earnings can non-parametrically identify the latent Markov process in firm and worker labor productivity. In my model, workers experience changes in their match specific labor productivity and their leisure preferences, and five periods of data on earnings and hours provide two variables to identify the two shocks. I leave a more formal proof for future work.

cause productivity shocks have a larger effect on output, it should be more difficult for firms to provide insurance against them as a result of limited commitment. Thus, productivity shocks should lead to more earnings volatility. Additionally, workers can escape bad job-specific labor productivity shocks by switching firms or switching to non-employment, so the job-specific labor productivity shock only affects the value of the current match. However, a leisure preference shock follows the worker into non-employment and to other firms, so it will impact both the current match and the worker's outside option values in similar ways. This allows for the contract to provide more insurance against preference shocks compared to productivity shocks, so preference shocks should cause less earnings variability compared to productivity shocks.

The probabilities that workers contact new firms when non-employed or employed in the model are given by λ_n and λ_e, respectively. In order to estimate these parameters, I target the fraction of employed workers who switch to a new firm (EE-rate) and the fraction of non-employed workers who become employed in the following period (NE-rate). In order to match the fraction of employed workers who become non-employed (NE-rate), I additionally include a probability δ that job-specific labor productivity enters an absorbing state of zero. When this happens, the firm and worker will certainly separate because the match will never be productive again. However, workers may transition to non-employment because of a high leisure preference shock or low labor productivity shock as well.

When workers contact a new firm when searching in the labor market, they draw an initial job-specific labor productivity $\hat{x}$ from a distribution F_0. Computationally, the job-specific labor productivity can only take a finite number of values from the Rouwenhorst approximation with $N_x = 5$ points. I assume that the distribution F_0 assigns equal probability to each of these points.

The final parameter in the contracting model is the value of home production b that workers consume when non-employed. A larger value of home production b will result in a larger value of non-employment and give workers better outside options. This will make it more likely that the worker's commitment constraint will bind when hours are high, requiring firms to pay higher earnings when hours are long. A larger value of b will also reduce the profits that firms earn because workers will have better outside options in bargaining. This means that firms' commitment constraint will also be more likely to bind, and workers will have to accept lower earnings when hours are reduced. Thus, the value of home production b will impact the covariance of changes in earnings and hours, and I target the elasticity of earnings with respect to persistent changes in hours.

In addition to the parameters in the structural model, I also allow for earnings and hours reported in the SIPP to contain measurement error ξ that is classical in logs with respect to true earnings and hours:

$$\log(\text{earnings}_{it}^{\text{report}}) = \log(\text{earnings}_{it}^{\text{true}}) + \xi_{iet}, \tag{30}$$

$$\log(\text{hours}_{it}^{\text{report}}) = \log(\text{hours}_{it}^{\text{true}}) + \xi_{iht}. \tag{31}$$

The measurement errors (ξ_{iet}, ξ_{iht}) each have mean zero, variances of $(\sigma^2_{\xi e}, \sigma^2_{\xi h})$, and have correlation coefficient ρ_ξ with each other. However, they are uncorrelated with the true values of earnings and hours and serially uncorrelated. As I show in Appendix C.1, it is necessary to target the auto-covariance over four periods to identify a permanent, persistent, and transitory component of an observed stochastic process. In order to estimate these parameters, I target the auto-covariance of earnings and hours across four periods as well as the covariance of changes in earnings and hours.

4.4 Estimation Procedure and Results

There are 17 parameters χ to be estimated:

$$\chi = \{\mu_\psi, \sigma_\psi, \mu_\theta, \sigma_\theta, \rho_{\psi\theta}, \quad \rho_x, \sigma_x, \rho_\phi, \sigma_\phi, \quad \lambda_n, \lambda_e, \delta, \quad b, \quad \sigma_{\xi e}, \sigma_{\xi h}, \rho_\xi\}. \tag{32}$$

I estimate the model by simulated method of moments to select the parameters χ to minimize the distance between moments simulated by the model M_{sim} and corresponding moments measured in the data M_{data} with a weight matrix W:

$$\chi^* = \arg\min_{\chi} (M_{\text{sim}}(\chi) - M_{\text{data}})' W (M_{\text{sim}}(\chi) - M_{\text{data}}). \tag{33}$$

Although the optimal weight matrix W would be the inverse of the variance-covariance matrix of the moments, this can lead to biases in finite samples (Altonji and Segal [1996]). Instead, I set the weight matrix to be the inverse of the diagonal of the variance-covariance matrix. The optimization proceeds by differential evolution (Storn and Price [1997]).

Results of the estimation are shown in Table 8, and the model fit is shown in Table 9. The model is broadly able to match the first and second moments of the cross-sectional distribution of earnings and hours relatively well even with only nine individual types. It is also able to match well the average job mobility rates. Although the NE-rate in the data and the estimated model is about 3.6%, the exogenous job destruction rate δ near zero. Thus, almost all separations to non-employment happen endogenously as a result of the labor productivity shocks x and the leisure preference shocks ϕ. The non-employed home production b is about 30% of average earnings for workers. This is consistent with

Table 8: Estimation Results

	Parameter	Estimates
1.	Mean Productivity Type (μ_ψ)	1.168
2.	S.D. Productivity Type (σ_ψ)	0.320
3.	Mean Preference Type (μ_θ)	-10.275
4.	S.D. Preference Type (σ_θ)	0.979
5.	Corr. Productivity and Preference Type ($\rho_{\psi\theta}$)	-0.920
6.	Auto-Corr. Productivity Shock (ρ_x)	0.001
7.	S.D. Productivity Shock (σ_x)	0.677
8.	Auto-Corr. Preference Shock (ρ_ϕ)	0.856
9.	S.D. Preference Shock (σ_ϕ)	0.185
10.	Non-employed Job Finding Rate (λ_n)	0.733
11.	Employed Job Finding Rate (λ_e)	0.154
12.	Exogenous Job Destruction Rate (δ)	0.001
13.	Non-employed home production (b)	38.710
14.	S.D. Meas. Error for Earnings ($\sigma_{\xi e}$)	0.330
15.	S.D. Meas. Error for Hours ($\sigma_{\xi h}$)	0.042
16.	Corr. Meas. Error for Earnings and Hours (ρ_ξ)	0.754

Note: This table shows the parameters estimated by simulated method of moments as described in Section 4.3.

estimates from previous literature looking at the change in earnings for workers entering non-employment in the SIPP and PSID (Rothstein and Valletta [2017], Braxton et al. [2024]).

The variability and persistence of earnings and hours in the model and the data match well. The persistence for the job-specific labor productivity shock process is estimated to be near zero. This is likely because of the high volatility of earnings observed in the SIPP across surveys that is highly correlated with hours. In order to rationalize this volatility, the model requires a volatile productivity shock process. However, a greater persistence in productivity would lead to a lower volatility of earnings in the model than observed in the SIPP data. This impersistence of productivity shocks is consistent with the work of Postel-Vinay and Turon [2010], who find that a serially uncorrelated productivity shock process and long-term contracts are sufficient to rationalize the dynamics of earnings in British survey data.[12]

[12] In contrast, Balke and Lamadon [2022] and Souchier [2022] estimate persistence of productivity to be greater than 0.9 using value added data from Sweden and France, respectively.

Table 9: Model Fit

	Moment	Model	Data
1.	$\mathbf{E}[\log e_{it}]$	5.075	5.075
2.	$\text{var}(\log e_{it})$	0.442	0.442
3.	$\mathbf{E}[\log h_{it}]$	3.356	3.356
4.	$\text{var}(\log h_{it})$	0.179	0.178
5.	$\text{cov}(\log e_{it}, \log h_{it})$	0.169	0.171
6.	EE Rate	0.062	0.061
7.	EN Rate	0.036	0.037
8.	NE Rate	0.365	0.365
9.	$\text{cov}(\log e_{it}, \log e_{it+l})$	0.305	0.297
10.	$\text{cov}(\log e_{it}, \log e_{it+2})$	0.276	0.280
11.	$\text{cov}(\log e_{it}, \log e_{it+3})$	0.262	0.267
12.	$\text{cov}(\log h_{it}, \log h_{it+l})$	0.084	0.077
13.	$\text{cov}(\log h_{it}, \log h_{it+2})$	0.071	0.073
14.	$\text{cov}(\log h_{it}, \log h_{it+3})$	0.066	0.069
15.	$\text{cov}(\Delta \log e_{it}, \Delta \log h_{it})$	0.050	0.049
16.	$\frac{\text{cov}(\Delta_{12} \log e_{it}, \Delta_{03} \log h_{it})}{\text{cov}(\Delta_{12} \log h_{it}, \Delta_{03} \log h_{it})}$	0.893	0.891

Note: This table shows the value of the parameters estimated by simulated method of moments as described in section 4.3. The variable h_{it} is the hours of work per week for individual i at time t, and e_{it} is earnings per week. Both variables are residualized by indicators for time, age, sex, race, education level, 10 industry groups, and 14 occupation groups.

Because the persistence of productivity shocks is so low, a highly persistent preference shock process is necessary to match persistence of hours in the data.

The estimates for the measurement error suggest that about 25% of variance of earnings is measurement error, while less than one percent of the variance of hours is measurement error. Empirically, there is much variability for earnings that is uncorrelated with changes in hours, and the model partly interprets this as measurement error. When linking SIPP earnings records to tax records, Gottschalk and Huynh [2010] estimates the variance of measurement error to be 27% of the total variance of earnings. However, they also report that much of the measurement error in the SIPP is mean reverting, and the variance of earnings in the SIPP is in fact lower than the variance of earnings in tax records. Similarly, Bound et al. [2001] report that much of the measurement error in survey data is mean reverting. Because the model does not allow for mean reverting measurement error, it likely understates the true importance of measurement error. Future work should consider how incorporating a mean reverting component to measurement error impacts the results of the estimation.

4.5 Heterogeneity in Earnings Variability in the Data and Model

In addition to matching the overall dynamics of earnings and hours in the data as shown in Table 9, the model also generates more earnings variability in employment contracts for workers with lower earnings, hours, and tenure as shown in the right panels of Figure 2. Earnings variability is measured by the average elasticity of earnings with respect to hours for workers across the quintiles of the earnings, hours, and tenure distribution in the model. This larger variability of earnings in employment contracts mirrors the empirical facts shown in Table 6 that hourly have lower earnings, hours, and tenure. Additionally, I show in the left panel of Figure 2 that workers in the SIPP in lower quintiles for earnings, hours, and tenure are more likely to be paid by the hour at their jobs. Thus, the model with risk-sharing implicit contracts constrained by limited commitment can explain why the variability of earnings in employment contracts is higher for workers with lower earnings, hours, and tenure.

The reason the model is able to generate this heterogeneity is because workers in the model are heterogeneous in the permanent labor productivity type ψ_i and permanent leisure preference type θ_i. Workers with higher permanent labor productivity types ψ_i or lower permanent leisure preference types θ_i will tend to have higher earnings, hours, and tenure at their jobs. Additionally, for these workers the outside option of non-employment is relatively less attractive compared to workers with lower permanent labor productivity types or higher permanent leisure preference types because all workers receive the same home production consumption b. Thus, they are less likely to hit their commitment constraint as a result of increases in hours in response to a shock, so their earnings remain more stable. Similarly, firms earn higher profits from these high permanent productivity type or low permanent leisure preference type workers, so they are less likely to hit their commitment constraint as a result of decreases in hours in response to a shock. As a result, workers with higher permanent labor productivity types or lower permanent leisure preference types will have more stable earnings in response to shocks compared to workers with lower permanent labor productivity types or higher permanent leisure preference types. This generates more earnings variability for workers with lower earnings, hours, and tenure in the model, which is consistent with heterogeneity in which workers receive hourly contracts.

5 Analysis

In this section, I use the estimated model to analyze the extent to which changes in labor productivity and leisure preferences pass through to worker's earnings, hours, and output as well as the implications of implicit contracts for restrictions on the variability.

5.1 Pass-through of Shocks

As shown in Proposition 2, if both firms and workers could fully commit to the implicit contract, earnings would be constant e_{i0} for each worker i and hours h_{it} would be set as:

$$\log h_{it} = \frac{1}{1-\alpha+\gamma}\left(\log x_{jt} - \log \phi_{it} - \tau \log e_{i0} + \log\left(\frac{\psi_i}{\theta_i}\right) + \log \alpha\right). \tag{34}$$

Here, ψ_i and θ_i are the permanent labor productivity and leisure preference types of worker i, while x_{jt} and ϕ_{it} are the job-specific labor productivity and leisure preference shocks. In the estimated model, the inverse Frisch elasticity of labor supply is set equal to 2 and the return to hours in production α is set equal to one. Thus, the elasticity of hours with respect to both productivity and preference shocks under full commitment would be 0.5, and the elasticity of earnings with respect to both shocks would be zero.

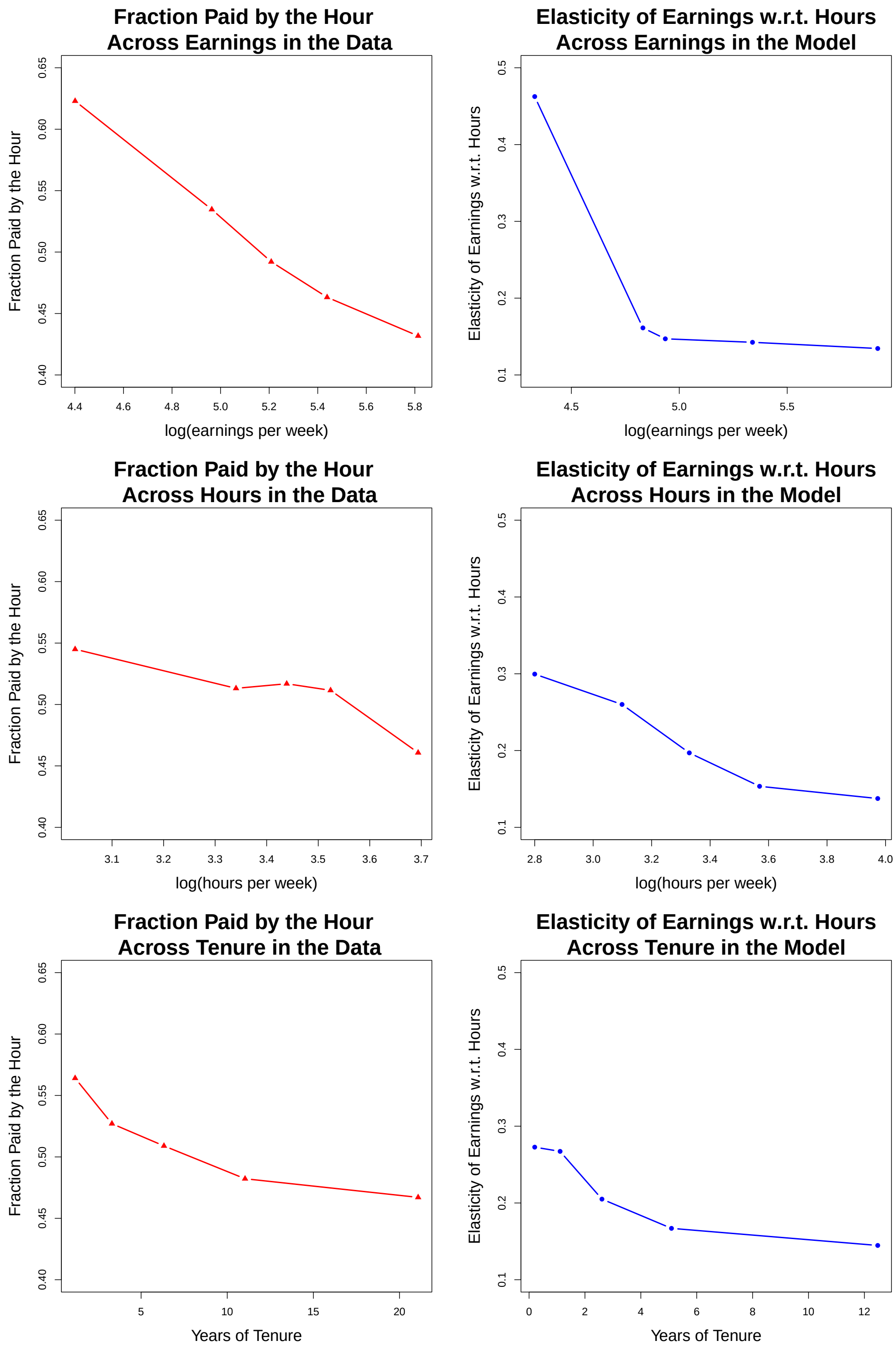
In the other extreme, if workers were able to set their own hours and were paid according to what they produced, then they would set their own hours as:

$$\log h_{it} = \frac{1}{1-(1-\tau)\alpha+\gamma}\left((1-\tau)\log x_{jt} - \log \phi_{it} + (1-\tau)\log \psi_i - \log \theta_i + \log \alpha\right). \tag{35}$$

Because the worker is risk-averse with coefficient of relative risk-aversion $\tau > 0$, the elasticity of hours with respect to shocks is smaller. In the estimated model, the coefficient of relative risk-aversion τ is set to one, so the elasticity of hours with respect to job-specific productivity shocks would be zero, and the elasticity of hours with respect to leisure preference shocks would be 0.333.

In the estimated model with implicit contracts constrained by limited commitment, the elasticity of hours and earnings with respect to the shocks will be somewhere between these two extremes as a result of limited commitment. The implicit contract will attempt smooth

Figure 2: Heterogeneity in Earnings Variability in Data and Model



Note: The left panels show the fraction of workers in the SIPP paid by the hour for quintiles of earnings, hours, and tenure. The right panels show the average elasticity of earnings with respect to hours in the model for quintiles of the earnings, hours, and tenure.

the workers earnings e_{it} across time and set hours h_{it} as:

$$\log h_{it} = \frac{1}{1-\alpha+\gamma}\left(\log x_{jt} + \log\phi_{it} - \tau\log e_{it} + \log\left(\frac{\psi_i}{\theta_i}\right) + \log\alpha\right). \tag{36}$$

However, earnings will vary in response to binding commitment constraints, and this will attenuate the elasticity of hours with respect to shocks. For instance, workers will threaten to quit their job if hours are so high and earnings so low that they would prefer to move to their outside option. As a result of this binding commitment constraint, the worker's earnings must be increased to keep them from leaving the match, and this will result in a reduction in the worker's hours relative to the optimal risk-sharing agreement.

To understand the actual pass-through of shocks in the estimated model, I run the following regression on workers' outcomes simulated from the estimated model:

$$\log(z_{it}) = \beta_i + \beta_x \log(x_{jt}) + \beta_\phi \log(\phi_{it}) + \epsilon_{it}. \tag{37}$$

Here, the outcome z_{it} is either hours, output, or earnings for worker i at time t, and β_i is a fixed effect for individual type. The estimated coefficients β_x and β_ϕ measure the pass-through of labor productivity and leisure preference shocks, respectively, to the outcome. Results are shown in Table 10.

Workers with a one percent higher preference shock have 0.416% lower hours, 0.416% lower output, and 0.167% lower earnings. These pass-through estimates show that firms provide partial insurance to workers against shocks to leisure preferences. On average, 60% of the change in output resulting from changes in leisure preferences is absorbed into firms' profits rather than being passed onto the workers' earnings. Relative to leisure preference shocks, however, firms provide more insurance to workers against changes in labor productivity. Workers with a one percent higher productivity shock have 0.423% higher hours, 1.423% higher output, and 0.154% higher earnings. This means that only 11% of the change in output resulting from changes in labor productivity shocks is passed-through to workers' earnings.

There are several factors influencing the extent to which firms can insure workers' earnings against shocks to labor productivity and leisure preferences. First, labor productivity shocks in the model are assumed to be job-specific, which means it affects the expected value of the current job but does not influence the values of workers' outside options. When labor productivity is low, workers will produce less at their current job and also be more likely to leave their current job for a new job with higher labor productivity. Thus, a labor productivity shock reduces firms profits not only in the current period by reducing

Table 10: Pass-through of Shocks to Workers' Outcomes

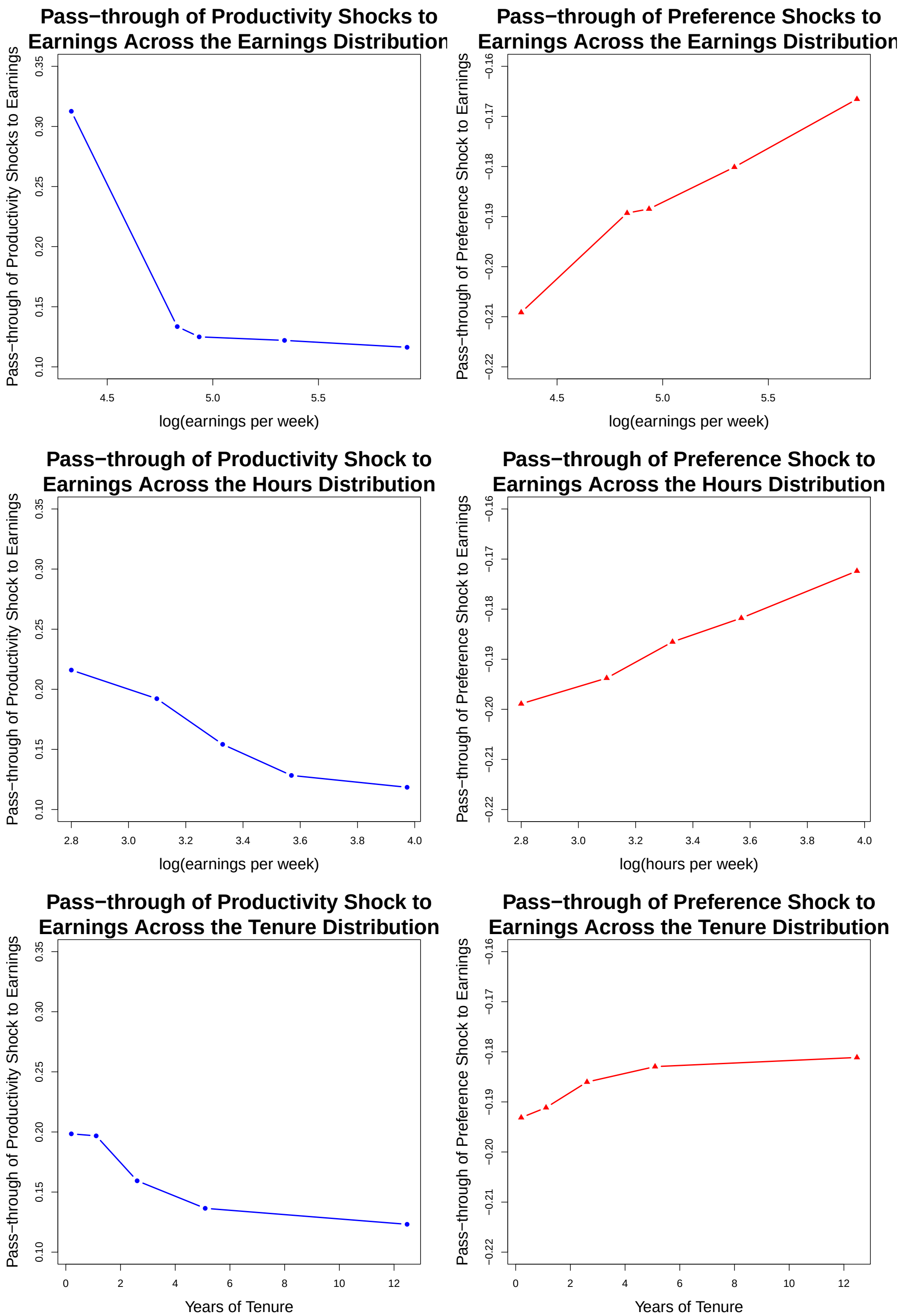
	(1) log(hours)	(2) log(output)	(3) log(earnings)
Productivity Shock $\log(x)$	0.423	1.423	0.154
Preference Shock $\log(\theta)$	-0.416	-0.416	-0.167
Individual Fixed Effect	Yes	Yes	Yes

Note: This table show how outcomes differ for workers with different shocks to preferences and productivity in simulations of the estimated model by estimating regression coefficients for equation 37.

output, it also reduces the expected future profits by changing the probability that the worker stays employed at a job. However, the second factor influencing the extent to which firms can insure workers in the implicit contract is the persistence of these shocks. The estimated leisure preference shock is highly persistent with an auto-correlation of 0.856, meaning that a worker with high leisure preferences today will also likely have high leisure preferences tomorrow. This reduces the firm's ability to insure workers' earnings against these shocks because firms profits will be reduced today and in future periods. In contrast, labor productivity shocks are estimated to have almost no persistence. Although workers who have low labor productivity shocks today will produce less, they are likely to revert to higher labor productivity levels in the next period.

As a result of heterogeneity in the extent to which limited commitment constrains risk-sharing, the pass-through of shocks to workers' earnings differs across heterogeneous workers in the model. Figure 3 shows how the average pass-through of productivity and preference shocks to earnings differs for workers across the earnings, hours, and tenure distribution. For workers with lower earnings, hours, and tenure, limited commitment is more likely to bind in their implicit contracts, and this generates larger pass-through of shocks to their earnings. For workers in the lowest earnings quintile, a one percent higher productivity shock would be associated with 0.32% higher earnings, while workers in the highest earnings quintile would have only 0.11% higher earnings. Similar patterns exist for preference shocks and across the distributions of hours and tenure.

Figure 3: Heterogeneity in the Pass-through of Productivity and Preference Shocks



Note: This figure shows how the pass-through of productivity and preference shocks in the estimated model differs on average for workers at quintiles of the earnings, hours, and tenure distributions.

5.2 Variance Decomposition

The results of the previous section show how changes in shocks pass-through to workers outcomes, but they do not measure the importance of these shocks for explaining different outcomes across workers. To understand the underlying sources of differences in earnings, hours, and output across workers, I perform a variance decomposition exercise in data simulated from the estimated model. To do this, I calculate for the regression in equation 37 the partial R-squared value for each explanatory variable: labor productivity shock x, leisure preference shock ϕ, and individual type i. The partial R-squared measures the fraction of the variance of the outcome that cannot be explained when that explanatory variable is removed from the regression. Results are shown in Table 11.

For earnings and hours, individual type i explains the largest portion of the variance. This is not surprising, because earnings and hours in the data are highly correlated across time for each individual. However, individual type explains substantially more of the variance in earnings compared to hours or output. The difference in the variance decomposition for earnings compared to hours arises in the model as a result of risk-sharing in the implicit contract. Changes in labor productivity and leisure preferences result in changes in hours, but the firm insures the worker's earnings against the resulting changes in output. Relative to leisure preference shocks, job-specific labor productivity shocks explain a larger fraction of the variance for all three outcomes. This is because the AR(1) process for job-specific labor productivity shocks is estimated to have a high variability. This generates sizeable differences in the job-specific labor productivity level.

5.3 Impact of Restricting Hours

In the implicit contract, hours play an important role to increase workers' total expected output by varying hours in tandem with labor productivity shocks. To quantify the importance of this mechanism, I restrict workers hours to depend only on their own leisure preferences as in equation 26 with the coefficient of relative risk-aversion τ set to one as in the estimated model:

$$h_{it} = \left(\frac{\alpha}{\phi_{it}\theta_i}\right)^{\frac{1}{1+\gamma}}. \tag{38}$$

In Table 12, I show how workers' outcomes in the model change compared to baseline when earnings adjust in two ways. In column 2, I adjust workers' earnings at fixed hourly wage rates. Under this scenario, workers benefit from the restrictions on hours by a consumption equivalence of \$14.34 per week on average (9% of average earnings per week). The restriction reduces the variability of their hours as a result of labor productivity shocks and also increases

Table 11: Partial R-Squared Variance Decomposition

	(1) log(hours)	(2) log(output)	(3) log(earnings)
Productivity Shock $\log(x)$	0.486	0.803	0.033
Preference Shock $\log(\theta)$	0.127	0.017	0.011
Individual Fixed Effects (i)	0.510	0.282	0.913
Individual Fixed Effect	Yes	Yes	Yes

Note: This table shows the partial R-squared for each outcome variable and explanatory variable in the estimated model. The partial R-squared for an explanatory variable measures the fraction of the variance for th outcome variable that cannot be explained by the other explanatory variables. The sum of partial R-squared values for a given outcome may sum to greater than 1 as a result of correlation between the explanatory variables.

workers' average earnings because workers now work longer hours on average. However, because hours are not able to vary in tandem with labor productivity shocks, workers' total expected output declines by 12%. This is because workers are now working longer hours when labor productivity is low and shorter hours when labor productivity is high. Because earnings adjusted at fixed hourly wage rates, this leads to lower average profits for firms, who go from receiving average profits of \$6.14 per person to losses of \$41.68 per person.

As a result of these losses at the restricted hours, firms will often prefer to fire their workers rather than continue the employment relationship. In order to keep their jobs, workers will have to accept a reduction in their earnings as a result of the restricted hours. In column 3 of Table 12, I show how workers' outcomes in steady state equilibrium of the model change when hours are restricted as in equation 38 but earnings are renegotiated in the employment contract in response to binding commitment constraints. Now, workers still benefit from having less variable hours as a result of the restriction, but the decline in their total expected output results in declines in their total expected earnings. When earnings in the contract adjust in response to the restriction, workers total expected earnings decline by 5.1%, and they are worse off by a consumption equivalence on average that is 5.2% of their baseline total expected earnings. Although workers benefit from having more stable hours from the restriction, the decline in output leads to declines in workers' earnings and employment rate that makes them worse off.

Table 12: Restricting Hours to Depend on Workers' Preferences

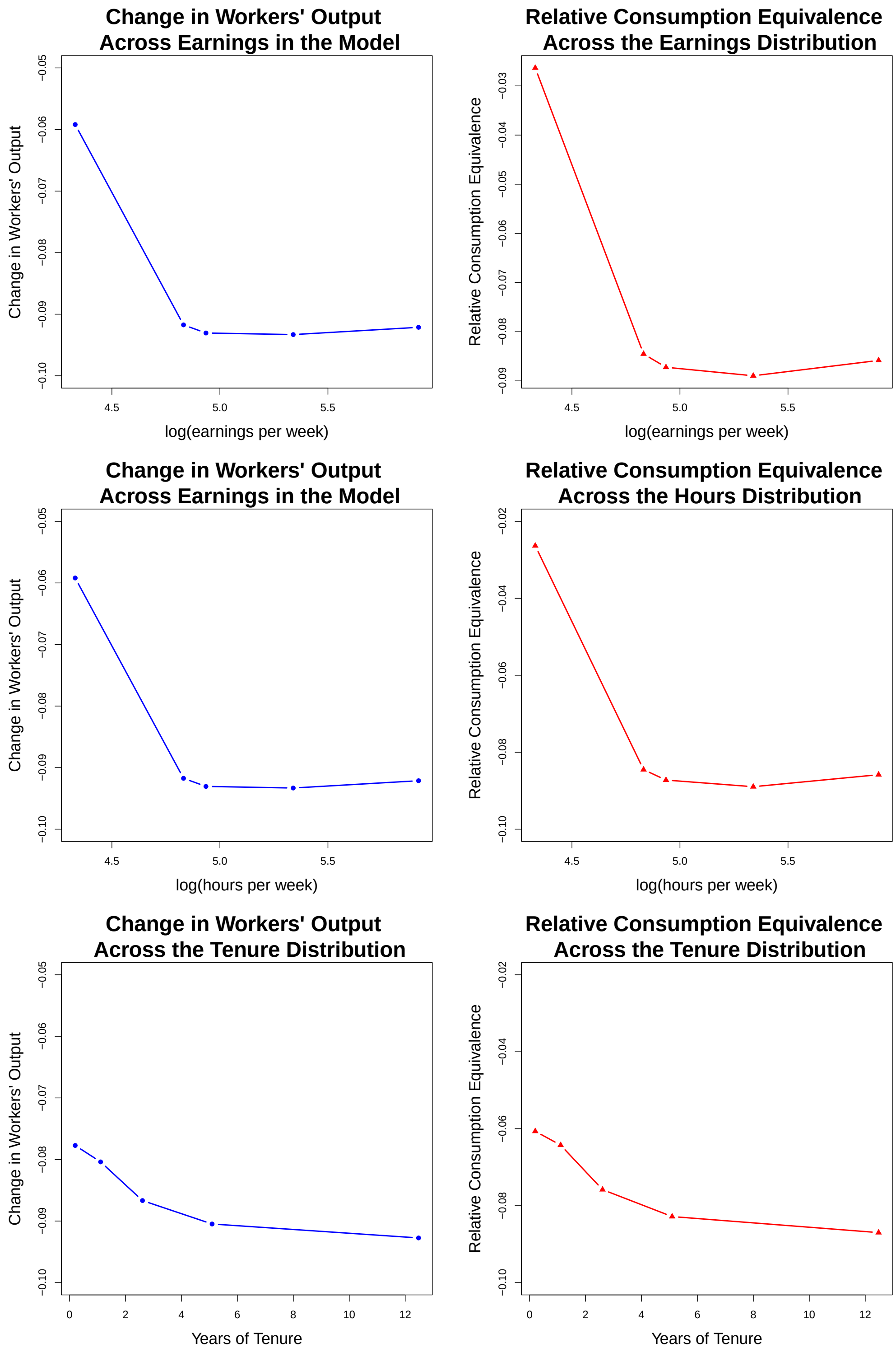
	(1) Baseline	(2) Earnings Change at Fixed Hourly Wage	(3) Earnings Change within Implicit Contract
Employment Rate	0.908	0.908	0.870
Average log(hours)	3.356	3.480	3.499
Variance Δ log(hours)	0.197	0.004	0.004
Average log(earnings)	5.075	5.200	5.025
Variance Δ log(earnings)	0.049	0.172	0.035
Output per Person	180.150	163.689	162.446
Profits per Person	6.144	-41.684	7.644
Workers' Consumption Equivalence ($ per week)	-	$14.335	-$8.404

Note: This table shows how workers' outcomes in the model change when hours are restricted to depend only on workers' preferences as discussed in Section 5.3. Consumption equivalence is defined as the transfer to all workers in each period of the hours restriction scenario that makes them indifferent on average with the baseline.

Because heterogeneous workers receive heterogeneous levels of insurance from firms in the implicit contracts, the costs of restricting hours in the contracting model is also heterogeneous. Workers who receive more insurance from firms will benefit more from hours variability and be harmed more when hours are not allowed to vary in tandem with labor productivity shocks. In Figure 4, I show how the change in output and consumption equivalence as a fraction of a workers' average earnings differs across the earnings, hours, and tenure distribution. Because workers with higher earnings, hours, and tenure receive more insurance from firms in implicit contracts, they also are harmed more when their hours are restricted in their employment contracts. While workers in the lowest earnings quintile are worse off by a consumption equivalence of 2.5% of their average earnings, workers in the highest earnings quintile are worse off by a consumption equivalence of 8.5% of their average earnings.

These results suggest that policymakers should be cautious when regulating the variability of hours in settings where firms and workers agree to implicit contracts. This is because the implicit contract optimally sets the variability of workers' hours to maximize the product of firms' total expected profits and workers' total expected values. When hours are restricted,

Figure 4: Heterogeneity in the Costs of Restricting Hours



Note: This figure shows how the costs of restricting hours variability as measured by changes in workers' output and relative consumption equivalence differs on average for workers at quintiles of the earnings, hours, and tenure distributions.

the implicit contract is unable to optimally set hours in response to shocks, and this leads to a reduction in workers' welfare. Current government policies regulating hours such as overtime policies and maximum hours restrictions often exempt certain classes of workers, and these workers are more likely to be in implicit contracts with their firms. In the United States, for example, workers whose job duties can be classified as executive, administrative, or professional are exempt from overtime pay requirements. The median manager in the SIPP data has 6.3 years of tenure at their current firm, and the median professional has 5.4 years of tenure. For workers who are not managers or professionals, median tenure is only 4.3 years. These differences in tenure suggest that managers and professionals are more likely to be in implicit (long-term) contracts with their firms and that exempting them from government policies regarding hours restrictions is beneficial. Similarly, recent predictive scheduling and fair workweek laws typically target only the hospitality and leisure industry, where median tenure in the SIPP is only 2.7 years. In all other industries, median tenure for workers is 5.1 years.

6 Conclusion

This paper documents empirically that changes in hours within a job are common, but the elasticity of earnings with respect to hours is larger for hourly workers compared to non-hourly workers. Using data from four consecutive surveys, I show that these patterns cannot easily be explained by classical measurement error. Instead, I find that the heterogeneity in the elasticity of earnings with respect to hours is consistent with a model of implicit contracts constrained by two-sided limited commitment. In an estimated model, firms provide substantial insurance to workers' earnings against shocks to labor productivity and leisure preferences, and hours play a quantitatively important role in amplifying labor productivity shocks to increase total expected output of workers. From the perspective of optimal implicit contracts, restrictions on hours reduce total expected output and welfare for workers. For workers with higher earnings, hours, and tenure, the benefits of implicit contracting and thus the costs of government regulations on hours are greater.

In the future, this work could be built upon in several dimensions. Empirically, improved data on earnings and hours for workers beyond survey data would provide a more accurate measurement of their joint dynamics. For earnings, it would be possible to relax the use of survey data from the Synthetic SIPP, a Census Bureau project that matches administrative income data with SIPP questions including hours worked. For hours, collecting and analyzing more accurate information for workers who are not paid by the hour is an important area for future work. The growing use of software such as Microsoft Teams and

Slack to coordinate work and schedule meetings for office workers may be fruitful for this purpose. Theoretically, the model could be extended to incorporate savings and coordinated production across workers. However, this coordination results in a non-concave production technology for hours that make solving for the optimal implicit contract theoretically challenging.

References

Daniel Aaronson and Eric French. The effect of part-time work on wages: Evidence from the social security rules. *Journal of Labor Economics*, 22(2):329–352, 2004.

John M Abowd and David Card. Intertemporal labor supply and long-term employment contracts. *The American Economic Review*, pages 50–68, 1987.

John M. Abowd and David Card. On the covariance structure of earnings and hours changes. *Econometrica*, 57(2):411–445, 1989.

Joseph G Altonji and Christina H Paxson. Labor supply, hours constraints, and job mobility. *Journal of Human Resources*, 27:256–278, 1992.

Joseph G Altonji and Lewis M Segal. Small-sample bias in gmm estimation of covariance structures. *Journal of Business & Economic Statistics*, 14(3):353–366, 1996.

Joseph G Altonji, Anthony A Smith Jr, and Ivan Vidangos. Modeling earnings dynamics. *Econometrica*, 81(4):1395–1454, 2013.

Enghin Atalay, Phai Phongthiengtham, Sebastian Sotelo, and Daniel Tannenbaum. The evolution of work in the united states. *American Economic Journal: Applied Economics*, 12(2):1–34, 2020.

Costas Azariadis. Implicit contracts and underemployment equilibria. *Journal of political economy*, 83(6):1183–1202, 1975.

Martin Neil Baily. Wages and employment under uncertain demand. *The Review of Economic Studies*, 41(1):37–50, 1974.

Neele Balke and Thibaut Lamadon. Productivity shocks, long-term contracts, and earnings dynamics. *American Economic Review*, 112(7):2139–77, 2022.

Paul Beaudry and John DiNardo. The effect of implicit contracts on the movement of wages over the business cycle: Evidence from micro data. *Journal of political Economy*, 99(4): 665–688, 1991.

Paul Beaudry and John DiNardo. Is the behavior of hours worked consistent with implicit contract theory? *The Quarterly Journal of Economics*, 110(3):743–768, 1995.

Andriana Bellou and Barış Kaymak. Wages, implicit contracts, and the business cycle: Evidence from a european panel. *Labour economics*, 19(6):898–907, 2012.

Alexander Bick, Adam Blandin, and Richard Rogerson. Hours and wages. *The Quarterly Journal of Economics*, 137(3):1901–1962, 2022.

Hans G Bloemen. Job search, hours restrictions, and desired hours of work. *Journal of Labor Economics*, 26(1):137–179, 2008.

John Bound, Charles Brown, Greg J Duncan, and Willard L Rodgers. Evidence on the validity of cross-sectional and longitudinal labor market data. *Journal of Labor Economics*, 12(3):345–368, 1994.

John Bound, Charles Brown, and Nancy Mathiowetz. Measurement error in survey data. In *Handbook of econometrics*, volume 5, pages 3705–3843. Elsevier, 2001.

J Carter Braxton, Kyle Herkenhoff, and Gordon M. Phillips. Can the unemployed borrow? implications for public insurance. *Journal of Political Economy*, 132(9):3025–3076, 2024.

Pierre Cahuc, Fabien Postel-Vinay, and Jean-Marc Robin. Wage bargaining with on-the-job search: Theory and evidence. *Econometrica*, 74(2):323–364, 2006.

Raj Chetty, John N Friedman, Tore Olsen, and Luigi Pistaferri. Adjustment costs, firm responses, and micro vs. macro labor supply elasticities: Evidence from danish tax records. *The Quarterly Journal of Economics*, 126(2):749–804, 2011.

Matthew S Dey and Christopher J Flinn. An equilibrium model of health insurance provision and wage determination. *Econometrica*, 73(2):571–627, 2005.

William T Dickens and Shelly J Lundberg. Hours restrictions and labor supply. *International Economic Review*, pages 169–192, 1993.

Greg J Duncan and Daniel H Hill. An investigation of the extent and consequences of measurement error in labor-economic survey data. *Journal of Labor Economics*, 3(4): 508–532, 1985.

Andrés Erosa, Luisa Fuster, Gueorgui Kambourov, and Richard Rogerson. Hours, occupations, and gender differences in labor market outcomes. *American Economic Journal: Macroeconomics*, 14(3):543–90, 2022.

Eugene F Fama. Time, salary, and incentive payoffs in labor contracts. *Journal of Labor Economics*, 9(1):25–44, 1991.

Xiaodong Fan, Ananth Seshadri, and Christopher Taber. Estimation of a life-cycle model with human capital, labor supply, and retirement. *Journal of Political Economy*, 132(1): 48–95, 2024.

Shigeru Fujita and Giuseppe Moscarini. Recall and unemployment. *American Economic Review*, 107(12):3875–3916, 2017.

Peter Ganong, Pascal Noel, Christina Patterson, Joseph Vavra, and Alexander Weinberg. Earnings instability. 2024.

Claudia Goldin. A grand gender convergence: Its last chapter. *American Economic Review*, 104(4):1091–1119, 2014.

Peter Gottschalk and Minh Huynh. Are earnings inequality and mobility overstated? the impact of nonclassical measurement error. *The Review of Economics and Statistics*, 92(2): 302–315, 2010.

John Grigsby, Erik Hurst, and Ahu Yildirmaz. Aggregate nominal wage adjustments: New evidence from administrative payroll data. *American Economic Review*, 111(2):428–471, 2021.

Sheldon E Haber and Robert S Goldfarb. Does salaried status affect human capital accumulation? *ILR Review*, 48(2):322–337, 1995.

John C Ham and Kevin T Reilly. Testing intertemporal substitution, implicit contracts, and hours restriction models of the labor market using micro data. *American Economic Review*, 92(4):905–927, 2002.

Daniel S Hamermesh. 12 million salaried workers are missing. *ILR Review*, 55(4):649–666, 2002.

Milton Harris and Bengt Holmstrom. A theory of wage dynamics. *The Review of Economic Studies*, 49(3):315–333, 1982.

Shulamit Kahn and Kevin Lang. Constraints on the choice of work hours: Agency versus specific-capital. *Journal of Human resources*, pages 661–678, 1992.

Karen A Kopecky and Richard MH Suen. Finite state markov-chain approximations to highly persistent processes. *Review of Economic Dynamics*, 13(3):701–714, 2010.

Claudio Labanca and Dario Pozzoli. Hours constraints and wage differentials across firms. Technical report, IZA Discussion Paper, 2022.

Marta Lachowska, Alexandre Mas, Raffaele Saggio, and Stephen A Woodbury. Work hours mismatch. Technical report, National Bureau of Economic Research, 2023.

David Lagakos and Guillermo L Ordonez. Which workers get insurance within the firm? *Journal of Monetary Economics*, 58(6-8):632–645, 2011.

Richard Layard, Guy Mayraz, and Stephen Nickell. The marginal utility of income. *Journal of Public Economics*, 92(8-9):1846–1857, 2008.

David S Lee, Justin McCrary, Marcelo J Moreira, and Jack Porter. Valid t-ratio inference for iv. *American Economic Review*, 112(10):3260–3290, 2022.

Jeremy Lise, Costas Meghir, and Jean-Marc Robin. Matching, sorting and wages. *Review of Economic Dynamics*, 19:63–87, 2016.

Albert Marcet and Ramon Marimon. Recursive contracts. *Econometrica*, 87(5):1589–1631, 2019.

John Nash. Two-person cooperative games. *Econometrica: Journal of the Econometric Society*, pages 128–140, 1953.

John Pencavel. The productivity of working hours. *The Economic Journal*, 125(589):2052–2076, 2015.

Fabien Postel-Vinay and Jean-Marc Robin. Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6):2295–2350, 2002.

Fabien Postel-Vinay and Helene Turon. On-the-job search, productivity shocks, and the individual earnings process. *International Economic Review*, 51(3):599–629, 2010.

Jesse Rothstein and Robert G Valletta. Scraping by: Income and program participation after the loss of extended unemployment benefits. *Journal of Policy Analysis and Management*, 36(4):880–908, 2017.

K.G. Rouwenhorst. Asset pricing implications of equilibrium business cycle models. *Frontiers of Business Cycle Research*, pages 294–330, 1995.

Leena Rudanko. Aggregate and idiosyncratic risk in a frictional labor market. *American Economic Review*, 101(6):2823–2843, 2011.

Lin Shao, Faisal Sohail, and Emircan Yurdagul. Are working hours complements in production? *Journal of Economic Dynamics and Control*, 153:104696, 2023.

Christian Sigouin. Self-enforcing employment contracts and business cycle fluctuations. *Journal of Monetary Economics*, 51(2):339–373, 2004.

Martin Souchier. The pass-through of productivity shocks to wages and the cyclical competition for workers. Technical report, Working Paper, 2022.

Martha Stinson. Technical description of sipp job identification number editing in the 1990-1993 sipp panels. *US Census. http://www. nber. org/sipp/1991/sipp9093jid. pdf*, 2003.

Rainer Storn and Kenneth Price. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization*, 11:341–359, 1997.

Jonathan Thomas and Tim Worrall. Self-enforcing wage contracts. *The Review of Economic Studies*, 55(4):541–554, 1988.

Jonathan P Thomas and Tim Worrall. Limited commitment models of the labour market. *Scottish Journal of Political Economy*, 54(5):750–773, 2007.

Emircan Yurdagul. Production complementarities and flexibility in a model of entrepreneurship. *Journal of Monetary Economics*, 86:36–51, 2017.

A Data Appendix

A.1 SIPP Sample Construction

Table 13: SIPP Sample Construction Steps

	(1) Hourly Sample	(2) Non-Hourly Sample
1. Positive Longitudinal Weight	342,421	300,649
2. Observed Next Three Surveys	228,020	200,578
3. No Employer Switches	134,300	135,723
4. No Second Job All Four Surveys	122,798	125,795
5. Non-imputed Earnings All Four Surveys	98,344	101,156
6. Non-topcoded Earnings All Four Surveys	97,826	94,468
7. Non-imputed Hours All Four Surveys	96,613	93,407
8. Does Not Report Hours Vary All Four Surveys	89,048	85,755
9. Non-zero Earnings All Four Surveys	88,708	85,451
10. Non-zero Hours All Four Surveys	88,604	85,397

Notes: This table shows the number of observations remaining after each sample construction step.

A.2 How are non-hourly workers paid?

The SIPP unfortunately only reports whether workers are paid by the hour or not. Non-hourly workers do not report the type of contract that they receive from their employer. However, a similar question in the Panel Survey of Income Dynamics (PSID) gives workers the option to choose fixed salary, hourly, commissions, and tips. I restrict the PSID sample to heads of households and their spouses in 2019 who are not self-employed. Additionally, I include only heads of households who are linked to an original PSID family. Table 14 shows

Table 14: Contract Types in the PSID

	Share of Answers
Salaried	0.400
Salary and Commissions	0.016
Paid by the Hour	0.523
Hourly and Tips	0.009
Hourly and Commissions	0.004
Other	0.047
Observations	5,269

Note: This table shows response to questions regarding payment scheme in the 2019 PSID for the main job heads of households and their spouses who are not self-employed. Observations are weighted using individual cross-section weights.

that hourly and salary contracts alone account for 92.3% of all employment contracts. For the set of workers who choose that they are not paid by the hour, 86.3% report that they are paid a salary alone.

A.3 SIPP Subjective Question about Hours Changes

To understand the factors generating this variation in work hours within a job, I use two sets of subjective questions in the SIPP shown in Tables 15 and 16. First, workers in the SIPP are asked if they worked part-time (1-34 hours) in any weeks (Sunday to Saturday) during the survey. Those who responded affirmatively are further asked to choose a reason for working part-time from a menu of options. For workers who usually worked full-time, this is a reduction in their hours of work. Second, the SIPP asks workers if they were absent from their jobs without pay for any weeks (Sunday to Saturday) during the survey, and if yes, why. For all workers regardless of their usual hours, this represents a reduction in their working time. Together, the answers to reasons for part-time work and unpaid absences reveal that both demand-side factors (layoffs, business conditions, and slack work) and supply-side factors (illness and family care) are important sources of variations in hours

Table 15: Why worked less than 35 hours in a week?

	Share of Answers
Worked any part-time weeks this wave?	
Yes	0.117
If yes, why?	
Could not find full-time job	0.019
Wanted to work part time	0.022
Injury	0.018
Illness	0.090
Chronic Health Condition	0.010
Taking Care of Family	0.035
Full-time work week less than 35 hours	0.026
Slack work or material shortage	0.126
Job sharing arrangement	0.001
On vacation	0.471
In school	0.009
Other	0.173
Sample	Usually Full-time Hours
Observations	486,160

Note:
This table shows hours variation is caused by both demand and supply-side facts. The sample includes individuals usually working full-time, and their answers to if and why they worked part-time for at least one week during the survey wave. Observations are weighted using SIPP weights.

within a job.

Table 16: Why absent from work without pay?

	Share of Answers
Any full week absences without pay this wave?	
Yes	0.046
If yes, why?	
On layoff	0.109
Slack work or business conditions	0.196
Own injury	0.030
Own illness	0.107
Pregnancy/childbirth	0.034
Taking care of children	0.021
On vacation/personal days	0.272
Bad weather	0.024
Labor dispute	0.004
New job to begin within 30 days	0.028
Job-sharing arrangement	0.002
Other	0.175
Sample	All workers
Observations	575,280

Note:
This table shows hours variation is caused by both demand and supply-side facts. The sample includes individuals with a job during wave, and their answers to if and why they were absent from their job without pay. Observations are weighted using SIPP weights.

A.4 Subgroup Analysis

Table 17: Variability of Earnings and Hours within a Job by Sex

	S.D. $\Delta \log(\text{hours/week})$	S.D. $\Delta \log(\text{earnings/week})$	Fraction Hourly	Observations
Female	0.237	0.282	0.528	86,622
Male	0.196	0.251	0.491	87,379

Note: S.D. abbreviates standard deviation. Sample includes individuals who do not switch firms. Observations are weighted using longitudinal weights.

Table 18: Variability of Earnings and Hours within a Job by Education

	S.D. $\Delta \log(\text{hours/week})$	S.D. $\Delta \log(\text{earnings/week})$	Fraction Hourly	Observations
No High School Degree	0.287	0.322	0.806	11,330
High School Degree	0.218	0.291	0.711	45,903
Some College	0.216	0.264	0.590	59,366
College Degree	0.211	0.251	0.255	38,554
More than College	0.228	0.255	0.105	18,848

Note: S.D. abbreviates standard deviation. Sample includes individuals who do not switch firms. Observations are weighted using longitudinal weights.

Table 19: Variability of Earnings and Hours within a Job by education

	S.D. Δ log(hours/week)	S.D. Δ log(earnings/week)	Fraction Hourly	Observations
No High School Degree	0.287	0.322	0.806	11,330
High School Degree	0.218	0.291	0.711	45,903
Some College	0.216	0.264	0.590	59,366
College Degree	0.211	0.251	0.255	38,554
More than College	0.228	0.255	0.105	18,848

Note: S.D. abbreviates standard deviation. Sample includes individuals who do not switch firms. Observations are weighted using longitudinal weights.

Table 20: Variability of Earnings and Hours within a Job by Age

	S.D. Δ log(hours/week)	S.D. Δ log(earnings/week)	Fraction Hourly	Observations
Ages 25 to 34	0.236	0.288	0.533	47,709
Ages 35 to 44	0.221	0.278	0.502	67,226
Ages 45 to 54	0.211	0.250	0.499	59,066

Note: S.D. abbreviates standard deviation. Sample includes individuals who do not switch firms. Observations are weighted using longitudinal weights.

Table 21: Changes in Hours within a Job by Industry

	S.D. Δ log(hours/week)	S.D. Δ log(earnings/week)	Fraction Hourly	Observations
Agriculture	0.373	0.346	0.516	1,520
Army	0.234	0.231	0.086	755
Construction	0.264	0.308	0.674	8,269
Education	0.270	0.305	0.464	42,916
Finance	0.155	0.223	0.338	11,745
Government	0.155	0.227	0.377	13,686
Information	0.187	0.245	0.380	8,470
Leisure	0.261	0.339	0.667	8,261
Manufacturing	0.182	0.236	0.634	28,716
Mining	0.249	0.294	0.592	846
Other	0.221	0.270	0.421	5,492
Professional	0.216	0.257	0.439	12,039
Trade	0.198	0.274	0.573	22,167
Transportation	0.225	0.248	0.591	9,119

Note: S.D. abbreviates standard deviation. Sample includes individuals who do not switch firms in two consecutive surveys. Observations are weighted using longitudinal weights.

Table 22: Changes in Hours and Earnings within a Job by Occupation

	S.D. $\Delta \log(\text{hours/week})$	S.D. $\Delta \log(\text{earnings/week})$	Fraction Hourly	Observations
Administrative Support	0.193	0.257	0.633	24,836
Army	0.235	0.227	0.087	745
Construction	0.276	0.326	0.791	6,844
Farming	0.327	0.418	0.589	1,040
Installation	0.177	0.237	0.699	7,458
Management	0.180	0.231	0.265	24,005
Production	0.213	0.282	0.841	16,555
Professional	0.224	0.254	0.288	53,713
Sales	0.211	0.280	0.452	9,105
Service	0.265	0.335	0.725	19,959
Transportation	0.270	0.290	0.745	9,741

Note: S.D. abbreviates standard deviation. Sample includes individuals who do not switch firms in two consecutive surveys. Observations are weighted using longitudinal weights.

A.5 Payment Schemes and Worker Characteristics Across Time

Why are some workers paid by the hour and other workers paid fixed salaries? As pointed out by Hamermesh ((2002)), there has been surprisingly little attempt to explain these differences in contract types either empirically of theoretically. The few papers that do consider the distinction between hourly and salaried jobs consider it to reflect the tasks of the job (Fama ((1991)), Haber and Goldfarb ((1995))). Conversely, highly skilled, non-routine, and professional jobs will be paid fixed salaries because hours are difficult to monitor and may not have a large impact on production.

However, Hamermesh ((2002)) argues that these theories provide only a partial understanding of the underlying differences between hourly and salaried jobs based on time series in job characteristics and payment schemes.[13] Using data Current Population Survey for years 1978 to 1997, he shows that the realized fraction of workers paid by the hour was largely stable and in fact grew slightly as shown in Figure 5. Extending the analysis to 2023, the fraction of workers paid by the hour again remained stable until the Covid-19 pandemic in 2020. This stability of the fraction of workers paid by the hour is surprising given changes in characteristics of US workers. Over the last several decades, US workers have become more educated, more experienced, and work in industries and occupations that typically were more likely to be paid by the hour.[14]

To show this formally, Hamermesh ((2002)) uses data for workers in base year $b = 1978$ to predict the probability that they are paid by the hour based on their observable characteristics in a linear probability model:

$$\text{Hourly}_{ib} = \beta_b X_{ib} + \epsilon_{ib} \tag{39}$$

[13] Hamermesh ((2002)) uses data from the CPS starting in the year 1978. However, CPS data in IPUMS only includes the variable for whether workers are paid by the hour starting in 1982, so I conduct my analysis using $b = 1982$ as the base year. The overall message of the Hamermesh ((2002)) is largely unaffected by this difference in base year.

[14] Atalay et al. ((2020)) additionally show that job tasks have changed over time within occupations, so the observable changes in industries and occupations understates the true change in tasks done by workers.

Here, Hourly_{it} is an indicator equal to 1 if the worker is paid by the hour, and X_{ib} is a set of observable characteristics in the base year including workers including age, education, race, region, sex, industry, and occupation. Using the estimated regression coefficients $\hat{\beta}_b$ from equation 39, Hamermesh ((2002)) then predicts the fraction of workers that should be paid by the hour in year t as:

$$\text{H}\overline{\text{ourly}}_t^* = \hat{\beta}_b \overline{X}_t. \tag{40}$$

Here, $\overline{X}_{it}$ is the average observable characteristics of workers in year t, and $\text{H}\overline{\text{ourly}}_t^*$ is the predicted fraction of workers paid by hour in year t.

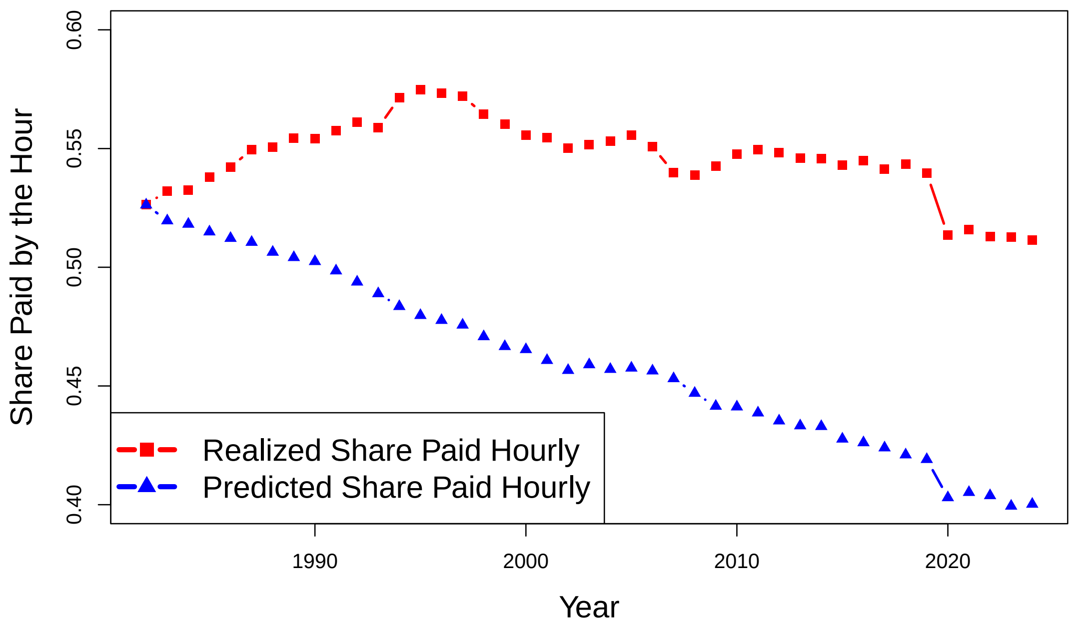
Results from this prediction are shown in Figure 5. Because of the changing characteristics of US workers, there should have been a decline in the number of workers paid by the hour. At the turn of the century, the fraction paid by the hour should have decline by about 7 percentage points, and by 2019 there should have been an additional 3 percentage point decline. However, as shown in the figure the realized fraction of workers paid by the hours has remained roughly constant and even grown slightly. This led Hamermesh ((2002)) to proclaim that there were "12 Million Salaried Workers are Missing" as the title of his paper in 2002 (p. 649). In 2024, that number has grown to over 17 million.

B Model Appendix

B.1 The Optimal Implicit Contract

Following Marcet and Marimon ((2019)), we can rewrite the contracting problem in equation 12 recursively with two additional state variables: Pareto weights on the worker and firm utility. I then show that we must only keep track of only the worker's earnings in the preceding period. Suppose that W^* is the initial value that the worker receives from the cooperative negotiation and consider how the firm would decide to optimally deliver this

Figure 5: Predicted and Realized Share of Workers Paid by the Hour



Note: This figure replicates and extends Hamermesh ((2002)). The red line with squares presents the realized fraction of workers paid by the hour in the Current Population Survey. The blue line with triangles presents the predicted fraction of workers that are paid by the hour as described in equations 39 and 40. Observations are weighted using longitudinal weights.

value to the worker in an implicit contract.[15] The firm's problem is:

$$\max_{\mathcal{C}} \ (1-d_t)\Big(\pi_0 + \mathbf{E}\Big[\sum_{t=1}^{\infty}\beta^t(1-D_t)(1-d_t)\pi_t \,\Big|\, \eta^0\Big]\Big), \tag{41}$$

subject to

1. Budget constraint at all histories η^t:

$$e_t + \pi_t = y(h_t|x_t). \tag{42}$$

2. Firm's commitment constraint at all histories η^t:

$$(1-d_t)\Big(\pi_t + \mathbf{E}\Big[\sum_{k=1}^{\infty}\beta^k(1-D_{t+k})(1-d_{t+k})\pi_{t+k} \,\Big|\, \eta^t\Big]\Big) \geq 0. \tag{43}$$

[15]It would alternatively be equivalent to define J^* to be the value that the firm receives from the cooperative negotiation and consider how the worker would deliver that value to the firm.

3. Worker's commitment constraint at all histories η^t:

$$(1-d_t)\Bigg(u(e_t,h_t|\phi_t)+\mathbf{E}\Big[\sum_{k=1}^{\infty}\beta^k\big\{(1-D_{t+k})(1-d_{t+k})u(e_{t+k},h_{t+k}|\phi_{t+k}) \\ +(1-D_{t+k})d_{t+k}\mathcal{B}^W_{t+k}\big\}\ \Big|\ \eta^t\Big]\Bigg)\geq(1-d_t)\hat{W}_t. \quad (44)$$

4. Worker receives cooperative negotiation value:

$$(1-d_0)\Bigg(u(e_0,h_0|\phi_0)+\mathbf{E}\Big[\sum_{t=1}^{\infty}\beta^t\big\{(1-D_t)(1-d_t)u(e_t,h_t|\phi_t) \\ +(1-D_t)d_t\mathcal{B}^W_t\big\}\ \Big|\ \eta^0\Big]\Bigg)\geq W^*. \quad (45)$$

This problem has the same set of constraints as the cooperative negotiation problem, so the solution to this problem will also be feasible for the cooperative negotiation game. The solution to this problem also will provide the value of the cooperative bargaining game to the worker, so it must also provide the value of the cooperative bargaining game to the firm. To see this, note that the contract that solves the cooperative negotiation game is feasible for this problem, so the firm must at least be better off here than they were in the cooperatively negotiated contract. However, if the firm was strictly better off here, then the cooperatively negotiated contract would not have been optimal. Thus, the contract solving the above problem is equivalent to the contract solving the cooperative negotiation game.

Let $\kappa_t^F(\eta^t)$ and $\kappa_t^W(\eta^t)$ be the Lagrange multipliers on the firm's and worker's commitment constraints at history η^t, respectively. Let μ be the Lagrange multiplier on the constraint that the worker initially receives a value W^* from the contract. Incorporating the constraints and Lagrange multipliers into the objective function, the firm's problem becomes:

$$
\begin{aligned}
\max_{\mathcal{C}} \ (1-d_0)\Bigg(&\pi_0 + \mu\big(u(e_0,h_0) - W^*\big) \\
&+ \mathbf{E}\Bigg[\sum_{t=1}^{\infty}\beta^t\Big\{(1-D_t)(1-d_t)\big(K_t^F\pi_t + K_t^W u(e_t,h_t) - \kappa_t^W \hat{W}_t\big) \\
&\qquad + (1-D_t)d_t K_{t-1}^W \mathcal{B}^W(\eta_t)\Big\}\ \Bigg|\ \eta^0\Bigg]\Bigg),
\end{aligned} \tag{46}
$$

such that for all histories η^t where the match has not yet been dissolved ($D_t = 0$), the budget constraint is satisfied:

$$
e_t + \pi_t = y(x_t, \psi, h_t), \tag{47}
$$

and the Pareto weights for the worker and firm update according to:

$$
K_t^W = K_{t-1}^W + \kappa_t^W, \tag{48}
$$

$$
K_t^F = K_{t-1}^F + \kappa_t^F. \tag{49}
$$

Here, K_t^F and K_t^W are the promised Pareto weights for the firm and worker in each period. Over time, the Pareto weights increase for a party whenever they have a binding commitment constraint. The term κ_t^P for party $P \in \{F, W\}$ determining this update is the Lagrange multiplier on the party's commitment constraint. If the constraint is not binding, then this will be zero and the Pareto weight will not update. When the constraint binds, this will be the minimum positive number that is needed to satisfy the constraint. The initial condition is that the worker must receive the value from the cooperative negotiation game. Therefore, the worker's Pareto weight in the first period will be equal to the Lagrange multiplier on this constraint ($K_0^W = \mu$). The firm's initial Pareto weight is one from the objective function, so $K_0^F = 1$.

With this recursive reformulation of the contracting problem in hand, we can characterize the optimal contract as a function of the promised Pareto weights. First, I show how to write the problem recursively with one additional state variable: the relative Pareto weight on the firm $K_t = K_t^F/(K_t^F + K_t^W)$. The firm's recursive problem with two additional state variables is:

$$V(\eta, K^F, K^W) = \max_{\mathcal{C}} \quad K^F_{new}\pi + K^W_{new}u(c,h) + \beta\mathbf{E}[V(\eta', K^F_{new}, K^W_{new})|\eta] - \kappa^W\hat{W} \tag{50}$$

subject to the same constraints as before. Note that we can define a new value function as

$$\tilde{V}(\eta, K) = V(\eta, K^F, K^W)/(K^F + K^W), \tag{51}$$

where $K = K^F/(K^F + K^W)$ is the relative Pareto weight on the firm. Dividing equation 50 by $K^F + K^W$ and substituting in 51, we arrive at:

$$\tilde{V}(\eta, K) = \max_{\mathcal{C}} \ (1 + \tilde{\kappa}^F + \tilde{\kappa}^W)(K'\pi + (1-K')u(c,h) + \beta\mathbf{E}[V(\eta', K')|\eta]) - \tilde{\kappa}^W\hat{W} \tag{52}$$

subject to the same constraints as before. Here, $\tilde{\kappa}^P = \kappa^P/(K^F + K^W + \kappa^F + \kappa^W)$ for $P \in \{F, W\}$, and $K' = (K + \kappa^F)/(1 + \kappa^F + \kappa^W)$.

From the first order conditions of this problem, we can characterize the optimal decisions in the contract:

$$e = \left(\frac{K'}{1-K'}\right)^{-\frac{1}{\tau}}, \tag{53}$$

and

$$h = \left(\alpha\frac{\psi_i}{\theta_i}\frac{x}{\phi}\frac{K'}{1-K'}\right)^{\frac{1}{1-\alpha+\gamma}} \tag{54}$$

Here, we see that decisions depend on both the promised relative Pareto weight and the shocks to productivity in the match. Consumption depends only on the relative Pareto weight. When neither firm nor worker commitment constraint bind, consumption is constant within a match. Consumption will increase in response to a worker's binding commitment constraint, while the opposite is true when the firm's constraint binds. Hours depend both on the Pareto weight and the productivity shock. A perfect correlation exists between hours and productivity when neither constraint binds. Binding commitment constraints for the worker decrease hours, and binding commitment constraints for firms increase hours. The optimal contract decision rules highlight why productivity shocks and two-sided limited commitment can generate hours variation in the data that is not proportional to earnings variation.

From here, we can prove Proposition 1. Equation 53 shows that there is a one-to-one mapping between the relative Pareto weights K' and the worker's earnings e. For each state $\eta = \{\phi, x, \hat{x}\}$, the worker has an outside option value of $\hat{W}$. There exists a relative Pareto weight K at which the worker's earnings and hours are set such that she is indifferent between staying in the contract and going to her outside option. This maps to the minimum earnings $e_{\min}(\eta)$ that the worker must be paid. Similarly, the firm's outside option value in each period is to receive total expected profits of zero. There exists a relative Pareto weight K at which point the firm receives total expected value from staying in the contract of zero. This maps to the maximum earnings that the contract is willing to pay the worker.

Because the outside option for the worker $\hat{x}$ is observed before decisions are made, the contract will only dissolve if there is no Pareto weight K where the firm and worker's commitment constraints are satisfied simultaneously. This will be the case when the relative Pareto weight K that makes the worker indifferent between staying in the match is smaller than the relative Pareto weight that makes the firm indifferent between staying in the match. This would imply that the minimum earnings that the worker needs to receive is larger than the maximum earnings that firm is willing to pay. In cases where the match does not dissolve, the relative Pareto weight will adjust by as little as possible in order to satisfy the binding commitment constraint. In the case where the worker's commitment constraint binds, this means that the worker will receive the minimum earnings level. In cases where the firm's commitment constraint binds, this means that the worker will receive the maximum earnings level. When commitment constraints do not bind for either party, then the relative Pareto weight K will not update, so the earnings for the worker today will be the same as earnings for the worker in the previous period. When the match does not dissolve, hours are also set according to equation 54.

For the case where there is full commitment in Proposition 2, decisions in the optimal contract can be solved for similarly except there is no commitment constraints 43 and 44. The contract problem is only constrained by the budget constraint in equations 42 and that the worker receives the cooperatively negotiated value in equation 45. Thus, the Lagrange multipliers κ_t^F and κ_t^W will be zero in every period. The relative Pareto weight throughout all periods of the match where the contract does not dissolve would be $K = 1/(1+\mu)$, where μ is the Lagrange multiplier that the worker receives the cooperatively negotiated value in equation 45. Thus, earnings will be constant through the match from equation 53 and hours will be set optimally as in 54.

C Estimation Appendix

C.1 Identification in a Simpler Model

In order to provide some intuition for the identification of the contracting model, I derive point identification in a model where workers set their own hours and are paid according to what they produce. Here, it is possible to identify the processes governing the dynamics of earnings and hours using four periods of data.

Consider a large number of workers indexed by i whose earnings e_{it} and hours h_{it} are observed over four periods t, $t+1$, $t+2$, $t+3$. The data set is then $\{(e_{it}, h_{it})_{s=t}^{t+3}\}_{i=1}^{N_i}$. Workers' preferences for consumption and hours are given by:

$$u_i(c_{it}, h_{it}|\phi_{it}) = \log(c_{it}) - \theta_i \frac{h_{it}^{1+\gamma}}{1+\gamma}, \tag{55}$$

and their production is:

$$y_{it} = \psi_i x_{it} h_{it}^{\alpha}. \tag{56}$$

Here, θ_i and ψ_i are permanent shocks (types) for the workers' preferences and production that are jointly log-normal:

$$\begin{bmatrix} \log \theta_i \\ \log \psi_i \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\theta \\ \mu_\psi \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & \sigma_{\theta\psi}^2 \\ \sigma_{\theta\psi}^2 & \sigma_\psi^2 \end{bmatrix} \right). \tag{57}$$

The preference shock ϕ_{it} and productivity shock x_{it} processes follow an AR(1) process with a normally distributed innovation term:

$$\phi_{it+1} = \rho_\phi \phi_{it} + \epsilon_{it}^{\phi} \qquad \epsilon_{it}^{\phi} \sim \mathcal{N}(0, \sigma_\phi^2) \tag{58}$$

$$x_{it+1} = \rho_x x_{it} + \epsilon_{it}^{x} \qquad \epsilon_{it}^{x} \sim \mathcal{N}(0, \sigma_x^2) \tag{59}$$

These shocks are independent of each other and independent of the permanent types. Additionally, I assume that these shocks have reached a stationary distribution across workers, so their expectations and variances are:

$$\mathbf{E}[\phi_{it}] = 0, \tag{60}$$

$$\mathbf{E}[x_{it}] = 0, \tag{61}$$

$$\text{var}(\phi_{it}) = \frac{\sigma_\phi^2}{1 - \rho_\phi^2}, \tag{62}$$

$$\text{var}(x_{it}) = \frac{\sigma_x^2}{1 - \rho_x^2}, \tag{63}$$

Suppose workers set their own hours, are paid according to what they produce ($e_{it} = y_{it}$), and consume their earnings ($c_{it} = e_{it}$). The first order condition for the optimal choice of hours yields the following equation for hours:

$$\log(h_{it}) = \frac{1}{1+\gamma}\log\theta_i + \frac{1}{1+\gamma}\log\phi_{it} + \frac{1}{1+\gamma}\log\alpha. \tag{64}$$

The budget constraint yields the following equation for earnings:

$$\log(e_{it}) = \log\psi_i + \log x_{it} + \alpha\log h_{it}. \tag{65}$$

Moreover, earnings $\tilde{e}_{it}$ and hours $\tilde{h}_{it}$ as observed by the econometrician contain classical measurement error that is serially uncorrelated:

$$\log\tilde{e}_{it} = \log e_{it} + m^e_{it} \qquad m^e_{it} \sim \mathcal{N}(0, \sigma^2_{me}), \tag{66}$$

$$\log\tilde{h}_{it} = \log(h_{it}) + m^h_{it} \qquad m^h_{it} \sim \mathcal{N}(0, \sigma^2_{mh}). \tag{67}$$

I assume that γ is known, so there are twelve parameters to be estimated.

$$\{\mu_\psi, \mu_\theta, \sigma^2_\psi, \sigma^2_\theta, \sigma^2_{\psi\theta}, \rho_x, \sigma^2_x, \rho_\phi, \sigma^2_\phi, \sigma^2_{me}, \sigma^2_{mh}, \alpha\}. \tag{68}$$

These parameters can be estimated using a method of moment estimator derive from the four observations of earnings and hours.

First, consider the auto-covariance of hours over the four periods:

$$\operatorname{cov}(\log\tilde{h}_{it}, \log\tilde{h}_{it+1}) = \frac{1}{(1+\gamma)^2}\sigma^2_\theta + \frac{\rho_\phi}{(1+\gamma)^2}\frac{\sigma^2_\phi}{1-\rho^2_\phi} \tag{69}$$

$$\operatorname{cov}(\log\tilde{h}_{it}, \log\tilde{h}_{it+2}) = \frac{1}{(1+\gamma)^2}\sigma^2_\theta + \frac{\rho^2_\phi}{(1+\gamma)^2}\frac{\sigma^2_\phi}{1-\rho^2_\phi} \tag{70}$$

$$\operatorname{cov}(\log\tilde{h}_{it}, \log\tilde{h}_{it+3}) = \frac{1}{(1+\gamma)^2}\sigma^2_\theta + \frac{\rho^3_\phi}{(1+\gamma)^2}\frac{\sigma^2_\phi}{1-\rho^2_\phi} \tag{71}$$

Using these three equations, we can sequentially solve for $\rho_\phi, \sigma_\theta^2$, and σ_ϕ^2 as:

$$\rho_\phi = \frac{\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) - \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2})}{\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1})}, \tag{72}$$

$$\sigma_\theta^2 = \frac{\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \rho_\phi \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1})}{1-\rho_\phi}, \tag{73}$$

$$\sigma_\phi^2 = \frac{1-\rho_\phi^2}{\rho_\phi} \left((1+\gamma)^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}) - \sigma_\theta^2 \right). \tag{74}$$

With these values in hand, we can use the equation for the variance of observed hours:

$$\text{var}(\log \tilde{h}_{it}) = \frac{1}{(1+\gamma)^2}\sigma_\theta^2 + \frac{1}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi^2} + \sigma_{mh}^2 \tag{75}$$

to solve for the variance of the measurement error in hours as:

$$\sigma_{mh}^2 = \text{var}(\log \tilde{h}_{it}) - \frac{1}{(1+\gamma)^2}\sigma_\theta^2 - \frac{1}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi^2}. \tag{76}$$

Next, the covariance between changes in earnings and hours can be used to estimate the returns to hours in production.

$$\text{cov}(\Delta \log \tilde{e}_{it}, \Delta \log \tilde{h}_{it}) = \alpha \text{var}(\Delta \log h_{it}). \tag{77}$$

This implies:

$$\alpha = \frac{\text{cov}(\Delta \log \tilde{e}_{it}, \Delta \log \tilde{h}_{it})}{\text{var}(\Delta \log h_{it}) - 2\sigma_{mh}^2}. \tag{78}$$

Then, we can solve for the average leisure preference level μ_θ using the average hours in the economy:

$$\mathbf{E}[\log \tilde{h}_{it}] = \frac{1}{1+\gamma}(\mu_\theta + \log(\alpha)). \tag{79}$$

Before pinning down the random processes underlying labor productivity, we first need to pin down the covariance of the permanent leisure preference type θ_i and permanent labor productivity type ψ_i. This can be accomplished using the covariance of observed earnings and hours:

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{h}_{it}) = \frac{1}{1+\gamma}\sigma^2_{\psi\theta} + \alpha \text{var}(\log h_{it}), \tag{80}$$

This yields:

$$\sigma^2_{\psi\theta} = (1+\gamma)(\text{cov}(\log \tilde{e}_{it}, \log \tilde{h}_{it}) - \alpha \text{var}(\log h_{it})). \tag{81}$$

Now, consider the auto-covariance structure of earnings:

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) = \sigma^2_\psi + \rho_x \frac{\sigma^2_x}{1-\rho_x} + \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}) + \alpha\sigma^2_{\psi\theta} \tag{82}$$

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2}) = \sigma^2_\psi + \rho_x^2 \frac{\sigma^2_x}{1-\rho_x} + \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) + \alpha\sigma^2_{\psi\theta} \tag{83}$$

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+3}) = \sigma^2_\psi + \rho_x^3 \frac{\sigma^2_x}{1-\rho_x} + \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) + \alpha\sigma^2_{\psi\theta} \tag{84}$$

Using these three equations, we can sequentially solve for ρ_x, σ^2_ψ, and σ^2_x as:

$$\rho_x = \frac{\dfrac{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+3})}{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2})} - \alpha^2 \dfrac{\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) - \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2})}{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2})}}{1 - \dfrac{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1})}{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2})} - \alpha^2 \dfrac{\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1})}{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2})}}, \tag{85}$$

$$\begin{aligned}\sigma^2_\psi = \frac{1}{1-\rho_x}\Bigg(& \text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2}) - \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \alpha\sigma^2_{\psi\theta} - \\ & \rho_x(\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) - \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}) - \alpha\sigma^2_{\psi\theta})\Bigg),\end{aligned} \tag{86}$$

$$\sigma^2_x = \frac{1-\rho_x^2}{\rho_x}\left(\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) - \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \alpha\sigma^2_{\psi\theta} - \sigma^2_\psi\right). \tag{87}$$

With these values in hand, we can use the equation for the variance of observed earnings:

$$\text{var}(\log \tilde{e}_{it}) = \sigma^2_\psi + \frac{\sigma^2_x}{1-\rho_x^2} + \alpha^2(\text{var}(\log \tilde{h}_{it}) - \sigma^2_{mh}) + \alpha\sigma^2_{\psi\theta} + \sigma^2_{me} \tag{88}$$

to solve for the variance of the measurement error in earnings as:

$$\sigma_{me}^2 = \text{var}(\log \tilde{e}_{it}) - \sigma_\psi^2 - \frac{\sigma_x^2}{1-\rho_x^2} - \alpha^2(\text{var}(\log \tilde{h}_{it}) - \sigma_{mh}^2) - \alpha\sigma_{\psi\theta}^2. \tag{89}$$

Finally, the average labor productivity type μ_ψ can be solved from average earnings:

$$\mathbf{E}[\log \tilde{e}_{it}] = \mu_\psi + \alpha \log \tilde{h}_{it}. \tag{90}$$

In the model estimated in the paper, earnings and hours will further be affected by the risk-sharing between firms and workers within the implicit contract. However, by estimating the contact rates for meeting firms when non-employed λ_n and employed λ_e as well as the exogenous job destruction rate δ by matching the empirical employment transition rates, we can hopefully infer how much risk-sharing firms and workers can do within the contract, and the back out the random processes underlying productivity and preferences.