Long-term Employment Contracts for Earnings and Hours

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Abstract

This paper examines how labor productivity and leisure preference shocks transmit to workers' earnings and hours in a frictional labor market model where firms offer long-term contracts. These contracts are motivated by risk-sharing and constrained by two-sided limited commitment. The model is estimated to match the dynamics of earnings, hours, and job mobility in the United States using data from the Survey of Income and Program Participation. Labor productivity and leisure preference shocks account for 26% and 13% of the cross-sectional variance in hours worked, respectively. Hours play a crucial role in the contract by amplifying the elasticity of output with respect to productivity shocks by 0.2. Due to limited commitment, firms only partially insure workers' earnings. Consequently, 15% and 10% of output changes caused by productivity and preference shocks are passed through to earnings, respectively.

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1 Introduction

Firms and workers care not only about the total number of hours worked but also the variability of those hours across time. From the worker's perspective, hours variability can represent flexibility to adjust their own schedule or take leave for personal reasons. However, it can also represent unpredictability driven by changing demands of her employer. Additionally, hours variability can cause financial instability for workers if changes in hours result in changes in pay. These concerns have motivated policies that promote flexibility by mandating paid leave as well as regulating unpredictability by penalizing schedule changes. Still, much of the research on hours focuses on the level of hours worked (Goldin [2014], Bick et al. [2022]). Less is known about how common changes in hours are within a job, whether these changes are driven by firms' business needs or workers' personal lives, and how these changes affect workers' earnings.

This paper makes four contributions. First, I document that workers commonly report changes in their hours within a job, but the elasticity of earnings with respect to hours differs across workers based on their employment contract. Second, I develop a frictional labor market model of how firms and workers agree to long-term risk-sharing contracts when firm labor productivity and worker leisure preferences vary over time. This framework yields closed-form solutions for earnings and hours, which I use to characterize how shocks to labor productivity and leisure preferences transmit to a worker's earnings and hours. Third, I estimate the model to match the empirical dynamics of earnings, hours, and job mobility. Finally, I use the estimated model to decompose the sources of changes in hours and the pass-through of shocks to earning, hours, and output.

Using the Survey of Income and Program Participation, I document that both hourly and non-hourly workers commonly report changes in their usual hours of work per week across four-month survey waves even when staying employed at the same firm. The average absolute value of the change in usual hours of work per week is 2.4 and 3.2 hours per week for hourly and non-hourly workers, respectively. Although hourly workers report smaller changes in hours, they also tend to work fewer hours. Relative to their previous hours of work, both hourly and non-hourly workers report changes in hours of similar magnitude. It is well known that survey data contains measurement error, particularly for hours of work (Bound et al. [2001]), but repeated sampling of the survey can help alleviate issues related to measurement error (Griliches and Hausman [1986]). Specifically for workers employed at the same firm in four consecutive surveys, changes in hours across the first and fourth survey are predictive of changes in hours across the second and third survey. This means that changes in hours are persistent and not driven entirely by measurement error. Previous work has documented that changes in hours are common for workers who are paid by the hour in firms' administrative records (Bound et al. [1994], Lachowska et al. [2023], Ganong et al. [2024]). Here, I document that non-hourly workers report changes in hours of similar magnitude as hourly workers in survey data that cannot easily be explained by measurement error.

Although changes in hours are common within a job for both hourly and non-hourly workers, the elasticity of earnings with respect to hours is larger for hourly workers. Across surveys, a one percent change in hours is associated with a 0.4% change in earnings for hourly workers but only a 0.1% change in earnings for non-hourly workers. Measurement error in hours will lead to a downward bias in these estimates, and temporary changes in hours may have a differential effect on earnings than persistent changes in hours. To account for this, I use information about changes in hours across four consecutive surveys to disentangle persistent changes from temporary changes in reported hours. For persistent changes in hours, hourly workers still experience a larger elasticity of earnings with respect to hours. A persistent one percent change in hours is associated with a one percent change in earnings for hourly workers but only a 0.4% change in earnings for non-hourly workers.

In order to understand how firms design employment contracts for their workers' earnings and hours, I develop a frictional labor market model with on-the-job search where risk-neutral firms offer long-term contracts for earnings and hours to their risk-averse workers. Within a match, the firm's labor productivity and the worker's leisure preferences each follow a Markov process and generate a motive for hours to change within a job. As a result of shocks and changes in hours, output will vary within a job. Because workers are risk-averse, they are willing to accept a reduction in their expected earnings in order to receive a sufficiently large reduction in the variation of their earnings. In contrast, risk-neutral firms are willing to increase the variability of their profits in order to increase their total expected profits. This double coincidence of wants creates an opportunity for firms and workers to both benefit by sharing risk arising from variations in labor productivity and leisure preferences. The long-term contract agreed upon by the firm and worker specifies how shocks transmit to the worker's earnings and hours while also responding to outside offers that the worker receives in the frictional labor market. Relative to recent papers studying long-term contracts motivated by risk-sharing (Rudanko [2011], Balke and Lamadon [2022], Souchier [2022]), this model additionally incorporates and hours margin of adjust and leisure preference shocks. Although previous papers have studied hours within a long-term contract, they typically consider variations in aggregate hours in response to changes in aggregate productivity over the business cycle (Sigouin [2004]). This paper in contrast focuses on idiosyncratic shocks to labor productivity and leisure preferences generating changes in earnings and hours within a job.

Using the ideas of Marcet and Marimon [2019], I derive the optimal history-contingent contract decisions for earnings and hours that reveal how a firm and worker optimally share risk. Relative to a worker's preferred hours choice, the firm *amplifies* the responsiveness of hours to productivity and preference shocks. The worker's earnings, however, are *insured* against the resulting output variation, which is absorbed by variations in the firm's profits. However, if firms and workers can freely walk away from a contract when given a better outside option, the extent to which the firm can amplify hours variations and insure the worker's earnings will be constrained by limited commitment as in Thomas and Worrall [1988]. When commitment constraints rarely bind, the optimal contract approximates one with full commitment where the worker receives smooth earnings payments and hours vary in response to changes in labor productivity or leisure preferences. If they frequently bind, then the worker's earnings will vary in tandem with hours in order to satisfy the commitment constraints. While previous literature has considered differing job tasks as a reason for different employment contracts (Fama [1991], Haber and Goldfarb [1995], Hamermesh [2002]), different contracts arise in this model as a result of different levels of risk-sharing within an employment contract because of commitment constraints.

Using simulated method of moments, I estimate the model to match the dynamics of earnings, hours, and job mobility observed in the SIPP. Using the estimated model, I find that 26% of the variance of hours in the data is generated by match-specific labor productivity shocks, and 13% is generated by worker-specific leisure preference shocks. This suggests that firms' business needs are the main driver of changes in hours within a job rather than workers' personal lives. Most of the remaining variance in hours is accounted for by heterogeneity in workers' typical preference and productivity levels. Additionally, hours play an important role in the contract to amplify the effect of labor productivity shocks on match output. If hours remained fixed, then a 1% increase in match productivity would be associated with a 1% higher match output. However, workers with 1% higher match productivity have 1.2% higher output as a result of working 0.2% more hours. Finally, I find that workers are well insured against changes in output resulting from changes in their leisure preferences and labor productivity. Workers with 1% higher labor productivity shocks have 1.2% higher output but only 0.173 higher earnings, implying that firms absorb about 85% of changes in output associated with changes in productivity. Similarly, workers with 1% higher leisure preference shocks have 0.239% lower output but only -0.023% lower earnings, implying that firms absorb 90% of output changes associated with changes in leisure preferences.

The rest of this paper is organized as follows. Section 2 presents an overview of the related literature. Section 3 provides an overview of the data facts regarding earnings, hours, and employment contracts. Section 4 presents the structural model of long-term contracts. Section 5 details the estimation, and section 6 discusses the analysis of the simulated model. Section 7 concludes.

2 Related Literature

This paper contributes to the literature on long-term employment contracts motivated by risk-sharing. Starting with Baily [1974] and Azariadis [1975], many papers consider how risk-neutral firms optimally pass-through shocks to their risk-averse workers' earnings or hourly wages. Several recent papers incorporate these optimal long-term contracts into equilibrium search models (Rudanko [2011], Balke and Lamadon [2022]). Most papers in this literature, though, consider only earnings or hourly wages while abstracting from hours of work decisions. Papers that do consider hours of work typically only consider reduced-form predictions of the models (Beaudry and DiNardo [1995], Ham and Reilly [2002], Sigouin [2004]). I contribute to this literature by documenting several facts about earnings, hours, and job mobility, which I argue are consistent with a search model where firms offer long-term contracts to workers for their earnings and hours constrained by two-sided limited commitment. I then estimate the model to decompose the sources of hours variation and provide an environment for counterfactual analysis.

Additionally, this paper contributes to the literature on the determinants of labor supply. Many previous papers view hours as a fixed feature of a job to rationalize empirical features of labor supply (Kahn and Lang [1992], Altonji and Paxson [1992], Dickens and Lundberg [1993], Bloemen [2008], Chetty et al. [2011]), but I highlight that hours frequently vary across time within a job for individual workers. Several recent papers have also documented the empirical hump-shaped relationship between hours and hourly wages (Yurdagul [2017], Bick et al. [2022], Labanca and Pozzoli [2022], Shao et al. [2022]); these papers argue that this is evidence of the non-linear returns to hours in the production technology. My paper considers the role that long-term employment contracts motivated by risk-sharing play in generating empirical patterns in earnings, hours, job mobility. The model allows for patterns in working time similar the data: (1) hours will vary within a job because of shocks to productivity and preferences, (2) workers will frequently report a desire to change their working hours because insurance provided by the firm distorts their hourly wage, and (3) the elasticity of earnings with respect to hours will differ across jobs and workers based on the extent to which limited commitment constrains the contract.

3 Empirical Dynamics of Earnings and Hours

This section documents three key data patterns regarding the dynamics of earnings, hours, and job mobility: (1) workers commonly report changes in their hours within a job that is not entirely transitory, (2) the elasticity of earnings with respect to hours differs based on workers' payment scheme, and (3) hourly workers have shorter tenure and higher turnover than observable similar non-hourly workers.

The main data source is the Survey of Income and Program Participation (SIPP) for years 1990 to 2013. I build on the sample construction codes of Gertler et al. [2020]. Each household in the SIPP is surveyed once each four months for three to five years. Studying hours for all workers necessitates the use of survey data because administrative records either contain no information about hours or accurate information only for workers who are paid by the hour. While self-reported hours in survey data contain measurement error (Bound et al. [2001]), I discuss how repeated observations in the SIPP can help to account for classical measurement error that is serially uncorrelated.

Relative to other surveys that interview households annually or biennially such as the NLSY and PSID, the SIPP between 1990 and 2013 allows for accurate measurement of employment dynamics at a higher frequency. Starting in 2014, the SIPP transitioned to an annual frequency. The CPS interviews households each month and asks about hours of work in every survey, but questions on labor earnings are only asked in the outgoing rotation surveys and March ASEC supplements that happen one year apart. Across outgoing rotation surveys and March ASEC supplements, it is also not possible to identify for most workers if their employer changed.[1]

To avoid dynamics in earnings and hours generated by education and retirement that are beyond the scope of this paper, I restrict the sample to individuals between the ages of 25 and 59. This paper focuses on how firms design employment contracts, so I further drop observations for workers who are self-employed. Table 1 shows summary statistics about workers in the sample. In each SIPP survey, employed individuals are asked about their usual hours of work on their job: "How many hours per week did you usually work at all activities at this job?". Respondents were told that if they have variable hours, they should report the "approximate average" of their actual hours per week for the weeks that they worked. In addition to hours, individuals are asked if they are paid by the hour and how much they were paid in each month during the survey wave.[2] Although information about

[1] The CPS primarily serves as an employment survey, and the lack of detailed information on income motivated the creation of the SIPP as its name suggests.

[2] Unfortunately, more detailed questions about how workers are paid is not included. However, the PSID includes a more detailed set of options from which workers can report their payment scheme. In the appendix

earnings is available for all months during the survey wave, the "seam effect" causes the majority of changes in monthly earnings to happen across waves rather than within waves. Following Gertler et al. [2020], I use only the information about earnings in the last month of the wave, which I divide by weeks worked in the last month to arrive at a measure of weekly earnings. I deflate earnings to May 2001 dollars using monthly PCE.

Table 1: Summary Statistics

	(1)	(2)
Avg. Earnings/week (2001 $)	$519.30	$942.81
	(360.77)	(617.98)
Avg. Hours/week	38.2	43.3
	(9.0)	(9.6)
Avg. Age	40.3	41.4
	(9.6)	(9.3)
Fraction Female	0.502	0.450
Fraction with Bachelor's Degree	0.127	0.498
Fraction with New Employer Next Survey	0.062	0.044
Fraction Non-employed Next Survey	0.055	0.028
Contract Type	Hourly	Not Hourly
Observations	459,629	410,164

Note: This table reports summary information about the sample of workers in the SIPP for years 1990 to 2013. Observations are weighted using cross-sectional weights. Standard deviations are in parentheses.

3.1 Workers commonly report changes in hours within a job.

How common are changes in hours within a job? Table 2 shows that across survey waves even when staying employed at the same firm, workers commonly report changes in their usual hours of work per week. This is true for both hourly and non-hourly workers. On average, the absolute value of the change in usual hours of work per week is 2.4 for hourly workers

A.1, I show that the vast majority of workers not paid by the hour report that they are paid fixed salaries.

and 3.3 for non-hourly workers. Although hourly workers report smaller changes in hours on average, they also tend to work fewer hours. To account for differences in the typical level of hours, I also report the average absolute value of the change in the logarithm of hours. This statistic is 0.075 for hourly workers and 0.078 for non-hourly workers, so the absolute value of the change in hours is approximately 7 to 8% for both sets of workers. Furthermore, these large average changes in hours are not generated solely by outliers; 21.6% of hourly workers and 31.5% of non-hourly workers report a change in their usual hours of at least five in absolute value. Lastly, these changes in hours are not driven by particular subgroups of workers. In appendix A.3, I show that workers commonly report changes in their usual hours of work when staying employed at the same firm regardless of their age, education, sex, industry, occupation, calendar month, or survey year.

Table 2: Changes in Hours within a Job

	(1)	(2)
$\lvert\Delta$ Usual Hours per Week$\rvert$	2.4	3.2
	(5.19)	(5.94)
$\lvert\Delta$ log(Usual Hours per Week)$\rvert$	0.075	0.077
	(0.203)	(0.189)
Fraction with $\lvert\Delta$ Usual Hours per Week$\rvert \geq 5$	0.218	0.310
Fraction with $\lvert\Delta$ Usual Hours per Week$\rvert \geq 10$	0.096	0.153
Contract Type	Hourly	Not Hourly
Observations	393,801	364,797

Note: This table reports summary information about changes in hours across surveys for the sample of workers employed at the same firm in two consecutive surveys. Observations are weighted using cross-sectional weights. Standard deviations are in parentheses.

When seeing these statistics, the first thought of course will be that these changes in hours could largely be explained by measurement error. Survey data is plagued by measurement error, particularly for hours of work (Bound et al. [2001]). Additionally, these changes could largely be transitory rather than persistent. Using four repeated observations of the same individual at the same job, however, it is possible to test whether changes in hours are persistent. Formally, suppose reported hours h_{it} for individual i at time t contain two components that are additively separable in logs:

$$\log h_{it} = p_{it} + m_{it}. \tag{1}$$

Here, p_{it} is a persistent component of reported hours with $\text{cov}(p_{it}, p_{it+k}) > 0$, and m_{it} is a transitory component that is serially uncorrelated and uncorrelated with the persistent component p_{it}. If there was classical measurement error in hours, then this would be captured by the transitory component m_{it}. However, m_{it} could also contain true temporary changes in hours. Across two surveys, it is not possible to know whether an observed change was caused by the persistent component p_{it} or the transitory component m_{it}.

However, using four consecutive surveys we can test whether there are any persistent changes. I will refer to the change in hours from periods t to $t+3$ as the outer change in hours, and the change in hours from periods $t+1$ to $t+2$ as the inner change in hours. First, note that the covariance between the inner and outer change in hours is equal to the covariance of the inner and outer persistent change in hours:

$$\begin{aligned}
&\text{cov}(\log h_{it+3} - \log h_{it}, \log h_{it+2} - \log h_{it+1}) \\
&\qquad = \text{cov}(m_{it+3} - m_{it}, m_{it+2} - m_{it+1}) + \text{cov}(p_{it+3} - p_{it}, m_{it+2} - m_{it+1}) \\
&\qquad\qquad + \text{cov}(m_{it+3} - m_{it}, p_{it+2} - p_{it+1}) = \text{cov}(p_{it+3} - p_{it}, p_{it+2} - p_{it+1}). \quad (2)
\end{aligned}$$

This is because the measurement error is serially uncorrelated and uncorrelated with the persistent changes in hours, so the finally three terms of the expand covariance are zero. Suppose that there are no persistent changes in hours, so $p_{it} = p_i$ is constant across time for each individual. Then the covariance of the inner and outer changes in hours would be zero by equation 2.

To test this hypothesis, we can regress the change in hours from $t+1$ to $t+2$ on the change in hours from t to $t+3$ for the sample of workers who are employed at the same firm in four consecutive surveys:

$$\log h_{it+2} - \log h_{it+1} = \beta \left(\log h_{it+3} - \log h_{it}\right) + \gamma + \epsilon_{it}. \quad (3)$$

If there are in fact no persistent changes in hours and the covariance between these two variables is zero, then the resulting regression coefficient β would be zero. However, as shown in Table 3, the coefficient β is positive and statistically significant at any reasonable threshold for both hourly and non-hourly workers. Overall, workers with a 1% increase in hours from periods t to $t+3$ on average have a 0.11% increase in hours from periods $t+1$ to $t+2$. The persistence of changes in hours is slightly larger for hourly workers than for non-hourly workers. This means that reported changes in hours are persistent across time and cannot entirely be explained by transitory factors such as classical measurement error.

Previous work using firms' administrative records has documented that changes in hours

Table 3: Relationship between Inner and Outer Change in Hours

	(1)	(2)	(3)
$\log(\text{hours})_{t,t+3}$	0.110	0.122	0.097
	(0.002)	(0.002)	(0.002)
Contract	All	Hourly	Not Hourly
Observations	345,714	173,877	171,837

Note: This table reports the regression coefficients from equation 2 using the sample of SIPP workers who are employed at the same firm in four consecutive surveys. Robust standard errors are presented in parentheses.

within a job are common for hourly workers. This was originally noted in the validation study of the PSID, which examined the administrative records of a single large manufacturing company: "Company records for hourly workers also showed surprising variability in work hours and earnings from one pay period to the next" ([Bound et al., 1994, p. 347]). Additionally, Lachowska et al. [2023] analyze the hours reported in administrative records from Washington state and report: "(M)uch of the variation in hours appears to be within a job over time, as opposed to resulting from fixed employer and worker effects" (p. 17-18). Finally, Ganong et al. [2024] find in a payroll processing company's records that: "in one quarter of months earnings change by at least 21%... Virtually all of this earnings volatility is driven by fluctuations in hours." (p. 2). Thus, it should not be surprising that hourly workers in survey data like the SIPP also report changes in their hours. In administrative records of hours, though, it is not possible to observe accurate information about the hours for workers who are not paid by the hour. A contribution of this paper is to document that non-hourly workers report changes in their hours in surveys of similar magnitude to hourly workers.

Many papers view hours as a rigid feature of a job. This view is useful for rationalizing why workers report dissatisfaction with their hours (Altonji and Paxson [1992], Dickens and Lundberg [1993]), the inelastic response of hours to labor income taxes (Chetty et al. [2011]), and most recently non-linearities in the cross-sectional relationship between hours and earnings (Yurdagul [2017], Bick et al. [2022], Shao et al. [2022]). However, changes in hours are common empirically. This motivates this paper to think about how firms and workers agree to changes in hours and earnings within a job.

3.2 The hours elasticity of earnings differs by contract type.

Both hourly and non-hourly workers commonly report changes in their hours within a job, but what is the elasticity of their earnings with respect to hours? To answer this question, I regress the change in the logarithm of workers' earnings e_{it} from periods $t+1$ to $t+2$ on the change in the logarithm of workers' hours $t+1$ to $t+2$ for individuals employed at the same firm in four consecutive surveys t to $t+3$:

$$\log e_{it+2} - \log e_{it+1} = \beta(\log h_{it+2} - \log h_{it+1}) + \gamma + \epsilon_t \tag{4}$$

Here, β is the elasticity of earnings with respect to hours, meaning that a one percent change in hours would be associated with on average a β percent change in earnings. When estimated by ordinary least squares, the coefficient β measures this elasticity for a typical change in hours that may either be transitory of persistent. Results of estimating this regression by ordinary least squares are shown in the first two columns of table 4. For hourly workers, a 1% increase in hours is associated with a 0.393% increase in earnings, while the for non-hourly workers this elasticity is only 0.117. This measure of the elasticity will measure a weighted average of the relationship between persistent changes in hours on earnings as well as the transitory changes in hours. If transitory changes in hours are largely measurement error, then the estimated elasticity will be biased toward zero.

To estimate the elasticity of earnings with respect to persistent changes in hours, we can use the outer change in hours $\log h_{it+3} - \log h_{it}$ as an instrument for the inner change in hours $\log h_{it+2} - \log h_{it+1}$ for the sample of workers employed at the same firm for four consecutive surveys. The outer change in hours will is assumed to be uncorrelated with the inner change in the transitory component of hours:

$$\text{cov}(\log h_{it+3} - \log h_{it}, m_{it+2} - m_{it+1}) = 0, \tag{5}$$

so the outer change satisfies the exogeneity condition for a valid instrument. Additionally, it will be correlated with the inner change in the persistent component of hours:

$$\text{cov}(\log h_{it+3} - \log h_{it}, p_{it+2} - p_{it+1}) > 0. \tag{6}$$

As shown in table 3, this is a relevant instrument with an F-statistic sufficiently large for valid statistical inference (Lee et al. [2022]). As shown in columns three and four of table 4, the elasticity of earnings with respect to hours is larger for persistent changes in hours. Hourly workers with a 1% persistent increase in hours have on average a 1.054% increase in earnings, while non-hourly workers have on average a 0.432% increase in earnings. If transitory changes in hours are entirely measurement error and have no effect on earnings, then these estimates would suggest that 62% and 72% of the variance for the changes in hours are generated by measurement error for hourly and non-hourly workers, respectively. However, if transitory changes in hours are partially real and have a true effect on earnings, then this will over-state the importance of measurement error.[3]

Thus, whether measured by ordinary least squares or as an instrument, hourly workers' changes in earnings are more correlated with changes in their hours on their jobs. This elasticity is over three times larger when estimated by ordinary least squares, and about two and a half times larger when estimated using the instrument. The elasticity for hourly workers is nearly one as would be expected for hourly workers below forty hours per week. Thus, although hourly and non-hourly workers experience changes in hours that are similar in magnitude, the relationship between changes in earnings and changes hours is much stronger for hourly workers.

One concern of course will be that changes in earnings and hours are mechanically more correlated for hourly workers by design of the survey. In the SIPP, earnings are measured at the monthly level. However, workers are allowed to report their earnings to the survey as paychecks, yearly, monthly, or weekly amount and have these converted to a monthly amount. Additionally, workers can choose to report an hourly wage and hours and have their monthly earnings calculated in that way. Hourly workers presumably would be more likely to choose the hourly calculation, while non-hourly workers would likely choose a different way of reporting earnings. However, the reason the survey is designed this way is to reduce measurement error in earnings by having all respondents report a monthly amount. Thus, these measured elasticities likely provide relevant information about how changes in hours correlate with changes in earnings. As shown in Ganong et al. [2024], hourly workers do have month-to-month earnings variability that is highly correlated with hours changes, while non-hourly workers largely have stable earnings month-to-month. Another concern is that non-hourly workers are more likely to receive bonuses that hourly workers. Although changes in non-hourly workers' earnings have less of an effect on their earnings today, it may have a larger effect on their earnings when bonuses are paid at the end of the year. However, Grigsby et al. [2021] reported that bonuses account for at least 10% of earnings for less than 10% of workers. Thus, although the coefficients for non-hourly workers are likely to be downward biased as a result of bonus, the fact that relatively few workers receive bonuses suggests that this would not lead to large changes.

[3] Duncan and Hill [1985] report that the measurement error of annual changes in hours has a variance that is 80% of the variance of true annual changes in hours.

Table 4: Elasticity of Earnings w.r.t. Hours by Payment Scheme

	(1)	(2)	(3)	(4)
$\log(\text{hours})_{t+2} - \log(\text{hours})_{t+1}$	0.393	0.117	1.054	0.423
	(0.005)	(0.005)	(0.039)	(0.047)
Estimation	OLS	OLS	IV	IV
First Stage F-Stat			3,001	1,831
Contract	Hourly	Not Hourly	Hourly	Not Hourly
Observations	173,877	171,837	173,877	171,837

Note: This table presents regression coefficients from equation 4 estimated by ordinary least squares (OLS) and instrument variable (IV). The instrument for the inner change in hours $\log(\text{hours})_{t+2} - \log(\text{hours})_{t+1}$ is the outer change in hours $\log(\text{hours})_{t+3} - \log(\text{hours})_t$. The sample includes workers who are employed at the same firm for four consecutive surveys t to $t+3$. Observations are weighted using cross-sectional weights in period t.

3.3 Payment Schemes and Worker Characteristics Across Time

Why are some workers paid by the hour and other workers paid fixed salaries? As pointed out by Hamermesh [2002], there has been surprisingly little attempt to explain these differences in contract types either empirically of theoretically. The few papers that do consider the distinction between hourly and salaried jobs consider it to reflect the tasks of the job (Goldfarb [1987], Fama [1991]). Conversely, highly skilled, non-routine, and professional jobs will be paid fixed salaries because hours are difficult to monitor and may not have a large impact on production.

However, Hamermesh [2002] argues that these theories provide only a partial understanding of the underlying differences between hourly and salaried jobs based on time series in job characteristics and payment schemes.[4] Using data Current Population Survey for years 1978 to 1997, he shows that the realized fraction of workers paid by the hour was largely stable and in fact grew slightly as shown in Figure 1. Extending the time frame to most recent years available of 2023, the fraction of workers paid by the hour again remained stable until the Covid-19 pandemic in 2020. This stability of the fraction of workers paid by the hour is surprising given changes in characteristics of US workers. Over the last several decades, US workers have become more educated, more experienced, and work in industries

[4] Hamermesh [2002] uses data from the CPS starting in the year 1978. However, CPS data in IPUMS only includes the variable for whether workers are paid by the hour starting in 1982, so I conduct my analysis using $b = 1982$ as the base year. The overall message of the Hamermesh [2002] is largely unaffected by this difference in base year.

and occupations that typically were more likely to be paid by the hour.[5]

To show this formally, Hamermesh [2002] uses data for workers in base year $b = 1978$ to predict the probability that they are paid by the hour based on their observable characteristics in a linear probability model:

$$\text{Hourly}_{ib} = \beta_b X_{ib} + \epsilon_{ib} \tag{7}$$

Here, Hourly_{it} is an indicator equal to 1 if the worker is paid by the hour, and X_{ib} is a set of observable characteristics in the base year including workers including age, education, race, region, sex, industry, and occupation. Using the estimated regression coefficients $\hat{\beta}_b$ from equation 7, Hamermesh [2002] then predicts the fraction of workers that should be paid by the hour in year t as:

$$\overline{\text{Hourly}}^*_t = \hat{\beta}_b \overline{X}_t. \tag{8}$$

Here, $\overline{X}_{it}$ is the average observable characteristics of workers in year t, and $\overline{\text{Hourly}}^*_t$ is the predicted fraction of workers paid by hour in year t.

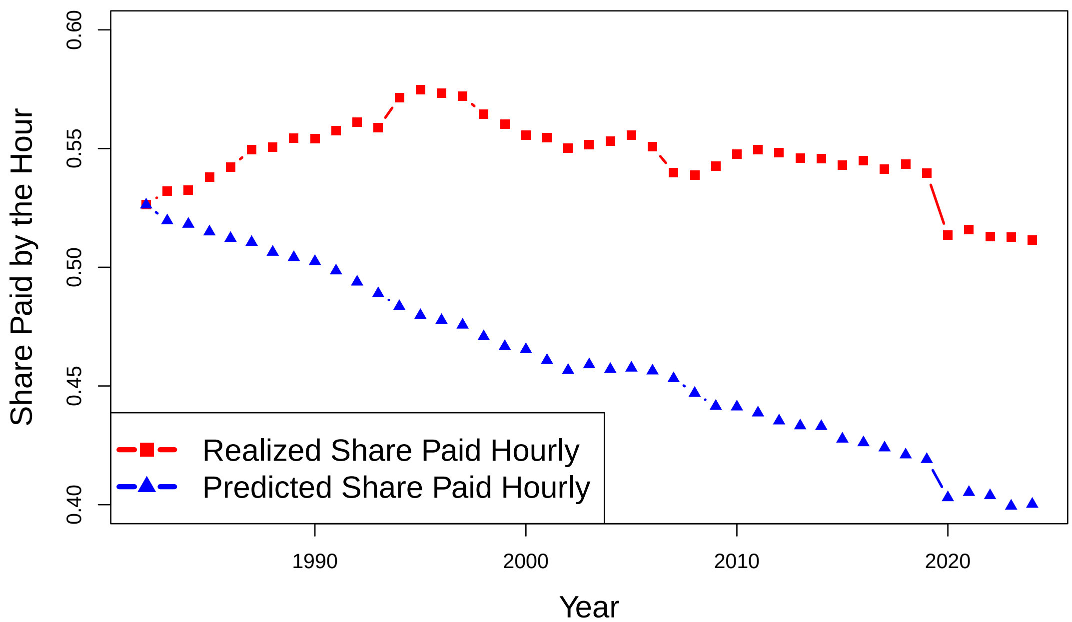
Results from this prediction are shown in Figure 1. Because of the changing characteristics of US workers, there should have been a decline in the number of workers paid by the hour. At the turn of the century, the fraction paid by the hour should have decline by about 7 percentage points, and by 2019 there should have been an additional 3 percentage point decline. However, as shown in the figure the realized fraction of workers paid by the hours has remained roughly constant and even grown slightly. This led Hamermesh [2002] to proclaim that there were "12 Million Salaried Workers are Missing" as the title of his paper in 2002 (p. 649). In 2024, that number has grown to over 17 million. Thus, in order to understand why some workers are paid by the hour and other workers are paid fixed salaries, we need to look beyond only observable characteristics of workers and jobs.

3.4 Hourly workers have shorter tenure and higher turnover

In his conclusion, Hamermesh [2002] hypothesizes that the distinction between hourly and non-hourly workers could represent a difference in trust between firms and workers rather than a difference in job tasks. The next section of this paper formalizes this view in a frictional labor market model with long-term contracts and limited commitment. To motivate this model, I provide one last set of empirical facts. Using the SIPP, I compare the tenure

[5] Atalay et al. [2020] additionally show that job tasks have changed over time within occupations, so the observable changes in industries and occupations understates the true change in tasks done by workers.

Figure 1: Predicted and Realized Share of Workers Paid by the Hour



Note: This figure replicates and extends Hamermesh [2002]. The red line with squares presents the realized fraction of workers paid by the hour in the Current Population Survey. The blue line with triangles presents the predicted fraction of workers that are paid by the hour as described in equations 7 and 8. Observations are weighted using cross-sectional weights.

and turnover of workers who are and aren't paid hourly in the following regression:

$$y_{it} = \beta \text{Hourly}_{it} + \gamma X_{it} + \epsilon_{it}. \tag{9}$$

Here, y_{it} is the outcome of interest, which is either the tenure of the worker at her job measured in years, an indicator equal to one if she switches to a new firm at the next survey, or an indicator equal to one if she is unemployed next survey. The vector X_{it} is a set of observable worker characteristics including education, age, sex, occupation, industry, the logarithm of earnings, the logarithm of usual hours per week, indicators for the survey year, and indicators for the survey month. The coefficient β measures the average difference in the outcome variable for hourly workers relative to observably similar workers who are not paid by the hour.

Results from the estimation of regression 9 are presented in table 5. Even after controlling for a rich set of observable worker characteristics, hourly workers have on average about three-fourths fewer years of tenure on their jobs, which is approximately 10% of the average

Table 5: Differences in Tenure and Turnover for Hourly and Non-Hourly Workers

	(1)	(2)	(3)
Hourly	-0.7441	0.0063	0.0049
	(0.0187)	(0.0006)	(0.0005)
Outcome	Tenure (Years)	Swith Firm Next Survey	Non-emp. Next Survey
Outcome Mean	7.6223	0.0527	0.0452
Observations	836,842	836,842	836,842

Note: This table presents regression coefficients on the indicator for hourly pay scheme from the regression in equation 9 using the sample of workers in the SIPP observed in two consecutive periods. Standard errors are in parentheses. The regression includes controls for age, sex, the logarithm of earnings, the logarithm of hours, occupation, industry, education, year, and month. Observations are weighted using cross-sectional weights.

tenure of the sample. Additionally, they are 0.6 percentage points and 0.4 percentage points more likely to transition to a new firm or non-employment at the next survey, respectively. Together, this means that hourly workers are about 10% more likely to leave their current firm than the average worker in the sample. This echoes one message from Ganong et al. [2024] that workers with more variable earnings who are typically paid by the hour are more likely to leave their jobs and motivates thinking about the joint dynamics of earnings and hours in a frictional labor market model.

4 Contracting Model

To understand the empirical dynamics of earnings hours, and job mobility, this section develops a model where firms offer long-term contracts and compete for workers in a frictional labor market. The long-term contracting framework builds on Thomas and Worrall [1988] by adding an intensive hours margin of adjustment, while the frictional labor market builds on Dey and Flinn [2005] by allowing firms to deliver values via long-term contracts. There are three key ingredients of the model to match the three empirical facts. First, a worker's hours will vary at her job as a result of variations in her leisure preferences or her firm's labor productivity. Second, her earnings will not necessarily vary in tandem with her hours or production as the result of risk-sharing in a long-term contract with her employer. Finally, two-sided limited commitment will constrain risk-sharing in long-term contracts and generate dynamics in earnings and hours that differ across workers. In particular, the elasticity of earnings with respect to hours will be higher for workers who are more likely to leave their jobs.

4.1 Agents and Environment

The labor market is frictional and in a steady state equilibrium. On one side of the market, there is a unit mass of workers who are heterogeneous in their average labor productivity ψ_i and average leisure preferences θ_i jointly distributed according to the distribution $F_{\psi\theta}$. On the other side of the market, there is a continuum of ex-ante identical firms who produce a homogenous good. Time t is discrete and runs forever. Both firms and workers live forever and discount the future with a common factor $\beta \in [0, 1)$. The probabilities that non-employed and employed workers contact a job at another firm are given by λ_n and λ_e, respectively.

4.2 Production and Preferences

Production is constant returns to scale across workers within a firm and across firms within the economy. Thus, the firm may consider the production of each worker independently of other workers' production. Output from a matched worker i and job j each period depends on the worker's productivity type ψ_i, a match-specific labor productivity shock x_{it}, and the worker's hours h_{it}. The production function is:

$$y_i(h_{it}|x_{it}) = \psi_i x_{it} h_{it}^{\alpha}. \tag{10}$$

Labor productivity shocks x_{it} follow a Markov process $f_x(x'|x)$, and the initial value of labor productivity at the start of a match is drawn from a distribution F_0. These shocks are match-specific and uncorrelated with shocks at other firms and to other workers. The parameter $\alpha \in (0, 1)$ measures the elasticity of production with respect to hours. Because there are diminishing returns to hours in production, output at a fixed labor productivity x_t would be maximized by smoothing hours across periods. When labor productivity varies over time, however, the firm can increase total expected production from the worker by requiring longer hours when productivity is high and shorter hours when productivity is low.

The flow utility to the worker each period depends upon her consumption c_{it}, hours h_{it}, preference type θ_i, and worker-specific leisure preference shock ϕ_{it}. Preferences are additively separable in consumption and hours and are represented by the utility function:

$$u_i(c_{it}, h_{it}|\phi_{it}) = \frac{c_{it}^{1-\tau} - 1}{1-\tau} - \theta_i \phi_{it} \frac{h_{it}^{1+\gamma}}{1+\gamma}. \tag{11}$$

Leisure preference shocks follow a Markov process $f_\phi(\phi'|\phi)$. These shocks are specific to a worker and will follow her across firms and into non-employment. The parameter $\gamma > 0$ captures the increasing marginal disutility from additional hours of work at a fixed leisure preference level ϕ_{it}. Although the worker would prefer to smooth her hours at a fixed ϕ_{it}, the firm can increase her total expected value from the match by requiring shorter hours when ϕ_{it} is high and longer hours when ϕ_{it} is low.

The parameter $\tau > 0$ measures the worker's desire to smooth her consumption across time. Lacking access to any financial instruments such as savings and borrowing, the worker will be hand-to-mouth and consume her labor earnings e_{it} when employed. When non-employed, the worker has zero hours of work and receives consumption b from home production that is common to all workers. Risk-aversion plus a lack of financial markets means that the worker would accept a reduction in her total expected earnings for a sufficiently large reduction in the variability of her earnings. In contrast to the worker, firms are risk-neutral and have deep pockets, so they desire only to maximize their total expected profits from the match with the worker. Risk-neutrality plus deep pockets imply that the firm would accept an increase in profit variability within the match in exchange for an increase in total expected profits from the match. Thus, there is a double coincidence of wants and opportunity for mutually beneficial trade under a long-term contract.

4.3 Long-Term Contracts and Limited Commitment

Consider a worker i starting a match at a time normalized to zero. In what follows, I drop the i subscripts for readability. The worker's leisure preference shock ϕ_t and match labor productivity x_t are observed by both the firm and worker each period prior to the choice of earnings and hours. Additionally, if the worker successfully contacts a new firm via search in the frictional labor market, the labor productivity that she draws at this firm $\hat{x}_t$ is observed by both the firm and worker prior to decisions. If the worker is unsuccessful in search, then $\hat{x}_t = \emptyset$ and her only outside option is non-employment. Let $\eta_t = \{\phi_t, x_t, \hat{x}_t\}$ be the current state variables, and let $\eta^t = \{\eta_0, ..., \eta_t\}$ be the history of shocks up through period t. Because the state of the world η_t is observed by both the firm and worker, it is possible to write a long-term contract $\mathcal{C}$ where labor earnings $e_t(\eta^t)$, hours of work $h_t(\eta^t)$, flow profits $\pi_t(\eta^t)$, and dissolution $d_t(\eta^t)$ are time-specific functions of the history of states η^t:

$$\mathcal{C} = \{e_t(\eta^t), h_t(\eta^t), \pi_t(\eta^t), d_t(\eta^t)\}_{t=0}^{\infty}. \tag{12}$$

For readability in what follows, I omit the functional argument for the decision variables.

However, both the firm and worker can each unilaterally dissolve the contract at any time for an outside option. Thus, the contract will be constrained by limited commitment and must be self-enforcing as in Thomas and Worrall [1988]. If either the firm or worker decides to dissolve the contract, they lose contact and cannot form a new match with each other in future periods. The ability of the firm to freely dissolve the contract is consistent with at will employment laws in all of the United States except Montana. For the worker, the freedom to dissolve the contract for an outside option firm is consistent with a lack of cumbersome non-compete clauses. When the match is dissolved by either party, the firm receives a terminal value of zero. The worker on the other hand receives a value either from moving to a new firm or to non-employment. Define $N_i(\phi)$ to be the value of non-employment to a worker i with leisure preference shock ϕ, and define $W_i^*(\phi, x)$ to be the highest value contract that a firm can offer a worker i with leisure preference shock ϕ when the initial labor productivity is x. Finally, define $\hat{W}$ to be the value of the worker's best outside option in period t:

$$\hat{W}_i(x, \phi) = \max\{N_i(\phi), W^*(\phi, \hat{x})\}. \tag{13}$$

If the worker does not meet another firm in period t, then $\hat{x} = \emptyset$ and $W_i^*(\phi, \emptyset) = \emptyset$. However, when moving to an outside firm the worker does not necessarily receive the value $W * (\phi, \emptyset)$. Instead, she must bargain for a contract. Define $\mathcal{B}_i^W(\phi, x, \hat{x})$ to be the contract that the worker i with leisure preference shock ϕ can bargain with a firm with labor productivity x when her outside option has labor productivity $\hat{x}$.

4.4 Contract Negotiation and a Recursive Formulation

The firm and worker cooperatively bargain the long-term contract $\mathcal{C}$ in order to maximize the product of their surplus values (Nash [1953]). Define $W(\mathcal{C}, \eta_0)$ and $J(\mathcal{C}, \eta_0)$ to be the values that worker and firm receive from contract $\mathcal{C}$. The cooperatively bargained contract solves:

$$\max_{\mathcal{C}} \quad \Big(W(\mathcal{C}, \eta_0) - \hat{W}_0\Big)\, J(\mathcal{C}, \eta_0), \tag{14}$$

subject to

1. Budget constraint:

$$e_t + \pi_t = y(h_t | x_t). \tag{15}$$

2. Firm's commitment constraint:

$$(1 - d_t)\Big(\pi_t + \mathbf{E}\big[\sum_{k=1}^{\infty} \beta^k (1 - D_{t+k})(1 - d_{t+k})\pi_{t+k} \;\big|\; \eta^t\big]\Big) \geq 0. \tag{16}$$

3. Worker's commitment constraint:

$$(1 - d_t)\Big(u(e_t, h_t|\phi_t) + \mathbf{E}\Big[\sum_{k=1}^{\infty} \beta^k \big\{(1 - D_{t+k})(1 - d_{t+k})u(e_{t+k}, h_{t+k}|\phi_{t+k})$$

$$+ (1 - D_{t+k})d_{t+k}\mathcal{B}^W_{t+k}\big\} \Bigm| \eta^t\Big]\Big) \geq (1 - d_t)\hat{W}_t. \quad (17)$$

Here, $D_t = \max\{d_k\}_{k=0}^{t-1}$ tracks whether either the firm or worker dissolves the contract prior to period t. Earnings for the worker will certainly be positive because of the functional form assumption for utility, but the flow profits to the firm π_t could be negative as long as the total expected profits in future periods are sufficiently positive as captured by the firm's commitment constraint. Similarly, the worker must receive a total expected value at each point in time that is weakly greater than her outside option value $\hat{W}_t$.

Although the optimal contract consists of an infinitely large set of functions that each depends on a set of arguments that grows ever larger over time, it is possible to recursively solve for the optimal contract decisions using the ideas of Marcet and Marimon [2019]. The optimal contract problem can be written as a recursive saddle point problem where decisions in period t depend not on the full history of states η^t but only the current state η_t and the previous periods earnings e_{t-1} as described in the following proposition.

Proposition 1. In any Pareto efficient contract, the following conditions are satisfied:

1. For all states of the world $\eta_t = (\phi_t, x_t, \hat{x}_t)$, there exists a lower bound on earnings $e_{\min}(\eta_t)$ and upper bound on earnings $e_{\max}(\eta_t)$.

2. The optimal decision in period t are:

   (a) The match dissolves if $e_{\max}(\eta_t) < e_{\min}(\eta_t)$:

$$d_t = \begin{cases} 1 & \text{if } e_{\max}(\eta_t) < e_{\min}(\eta_t), \\ 0 & \text{if } e_{\max}(\eta_t) \geq e_{\min}(\eta_t). \end{cases} \quad (18)$$

   (b) For all $t > 0$ when $d_t = 0$, earnings e_t are set according to:

$$e_t = \begin{cases} e_{\min}(\eta_t) & \text{if } e_{t-1} < e_{\min}(\eta_t), \\ e_{t-1} & \text{if } e_{\max}(\eta_t) \geq e_{t-1} \geq e_{\min}(\eta_t), \\ e_{\max}(\eta_t) & \text{if } e_{\max}(\eta_t) < e_{t-1}. \end{cases} \quad (19)$$

   (c) For all $t \geq 0$ when $d_t = 0$, hours h_t are set according to:

$$h_t = \left(e_t^{-\tau} \alpha \frac{\psi_i}{\theta_i} \frac{x_t}{\phi_t}\right)^{\frac{1}{\alpha+1-\gamma}}. \quad (20)$$

Proof: Appendix

In this proposition, the lower bound on earnings $e_{\min}(\eta_t)$ represents the lowest earnings that the contract must promise the worker in order to provide her value $\hat{W}_t$. Similarly, the upper bound on earnings $e_{\max}(\eta_t)$ represents the highest earnings that the contract can provide the worker while the firm receives non-negative total expected profits. If the lower bound on earnings is greater than the upper bound on earnings for a give state η_t, the firm's and worker's commitment constraints cannot be satisfied simultaneously, so they must separate; this is captured in equation 18. Otherwise, the contract will attempt to pay the worker the same earnings as the prior period; this is because the worker is risk-averse while the firm is risk-neutral, so an optimal risk-sharing agreement provides smooth earnings payments to the worker. When a change in state variables causes a binding commitment constraint at the previous earnings level, however, the contract adjusts the worker's earnings by as little as possible to satisfy the constraint. This adjustment impacts the worker's earnings not only today but also in future periods so long as commitment constraints do not bind. Although earnings for the worker will be smoothed across time, hours will adjust in response to shocks to changes in labor productivity x_t and leisure preferences ϕ_t. This is because correlating hours with productivity shocks increases the total expected output from the match, and correlating hours with leisure preference shocks increases the total expected value of the worker. When a commitment constraint binds, however, the firm optimally adjusts the worker's hours as well as her earnings to satisfy the constraint; this is captured by hours in period t depending upon earnings e_t in period t.

Because the chosen contract maximizes the product of the firm and worker surplus, they will certainly agree to a contract that is Pareto efficient. This means that there are no feasible contracts that can make the firm better off without making the worker worse off and vice versa. The above proposition casts the Pareto frontier in terms of the worker's initial earnings e_0. The minimum earnings $e_{\min}(\eta_0)$ represents the Pareto efficient contract where the firm earns the highest possible value, but the worker is indifferent between receiving a value of $\hat{W}_0$ and staying in the match. If earnings were any lower, then she would unilaterally dissolve the contract. Conversely, the maximum earnings represents the contract in which worker earnings the highest possible value $W^*(\phi_t, x_t)$, but the firm is indifferent between receiving a terminal value of zero and staying in the match. If earnings were any higher, then the firm would prefer to dissolve the contract. The worker's initial earnings e_0 will be decided to maximize the product of the firm's and worker's surplus values in equation 14

This proposition also provides a recursive formulation of the decisions in the contract. Optimal decisions in period t of the contract depend not on the full history of states up to period t, but only on the current state $\eta_t = \{\phi_t, x_t, \hat{x}_t\}$ and the previous period's earnings.

With this formulation in hand, we can define $W(e_{-1}, \phi, x, \hat{x})$ and $J(e, \phi, x, \hat{x})$ to be the value functions of the workers receive based on the previous earnings $e_{\text{-}1}$ and current state of the world:

$$W(e_{-1}, \phi, x, \hat{x}) = (1-d)\big(u\left(e, h|\phi\right) + \beta \mathbf{E}\left[W(e, \phi', x', \hat{x}')\right]\big) + d\mathcal{B}^W(\phi, \hat{x}, x) \tag{21}$$

$$J(e_{-1}, \phi, x, \hat{x}) = (1-d)\big(\pi + \beta \mathbf{E}\left[J(e, \phi', x', \hat{x}')\right]\big) \tag{22}$$

where decision d, e, and h are given in the Proposition 1. Here, the expectation operator is taken over the future states $(\phi', x', \hat{x}')$. In the initial period $t = 0$ of the contract, there are no earnings from period $t = -1$ because the worker was either employed at another firm or non-employed. Instead, the worker's earnings are initially set according to the cooperative negotiation to maximize the joint surplus values of the firm and worker.

$$e_0(\phi, x, \hat{x}) = \arg\max_{e} \;\; (W(e, \phi, x, \hat{x}) - \hat{W}(\phi, \hat{x}))J(e, \phi, x, \hat{x}) \tag{23}$$

Thus, solving for the optimal contract amounts to solving for the initial earnings level e_0 that maximizes the product of the surplus values received by the firm and worker. The value that the worker can receive from bargaining with a firm at state $(\phi, \hat{x}, x)$ is then:

$$\mathcal{B}^W(\phi, x, \hat{x}) = W(e_0(\phi, x, \hat{x}), \phi, x, \hat{x}). \tag{24}$$

4.5 Characterizing Risk-Sharing and Limited Commitment

The worker's hours are optimally set each period such that the worker's marginal rate of substitution of earnings for hours is equal to the marginal rate of transformation of hours into production:

$$\frac{u_c(e, h|\phi)}{u_h(e, h|\phi)} = y_h(h|x) \tag{25}$$

The marginal rate of transformation can also be thought of as the firm's marginal rate of substitution of earnings for hours where the firm's utility function is defined based on the budget constraint $\pi = y - e$. Note that the specific functional equations for utility and production are not necessary in order for an optimal long-term contract to exist. They are necessary, however, to easily characterize how hours respond to shocks in the equation.

To understand how risk-sharing and limited commitment influence the variation in earnings and hours, it is useful to consider two extreme cases: fully enforceable contracts and spot markets. With enforceable long-term contracts, the optimal contract does not need to adjust based on the firm's and worker's commitment constraints. It needs only to consider the worker's outside option value in the initial period $\mathcal{B}_0^W$ when setting earnings and hours as shown in the following proposition.

Proposition 2. If employment contracts were fully enforceable, then earnings are constant at all points in time in the match $(e_t(\eta^t) = e_0)$, and hours are set as:

$$h_t = \left(e_0^{-\tau} \alpha \frac{\psi_i}{\theta_i} \frac{x_t}{\phi_t} \right)^{\frac{1}{1-\gamma+\alpha}}. \tag{26}$$

Proof: Appendix.

Fully enforceable contracts mean that the contract can ignore binding commitment constraints in future periods. Earnings are set in the initial period to maximize the product of the firm's and worker's surplus values, and remain constant throughout the match because the worker is risk-averse. Hours each period optimally respond to shocks to labor productivity and leisure preferences. The elasticity of the worker's hours in response to shocks in fact does not depend at all upon her risk-averse level τ but only on the elasticity with respect to hours of production α and of utility $1-\gamma$.

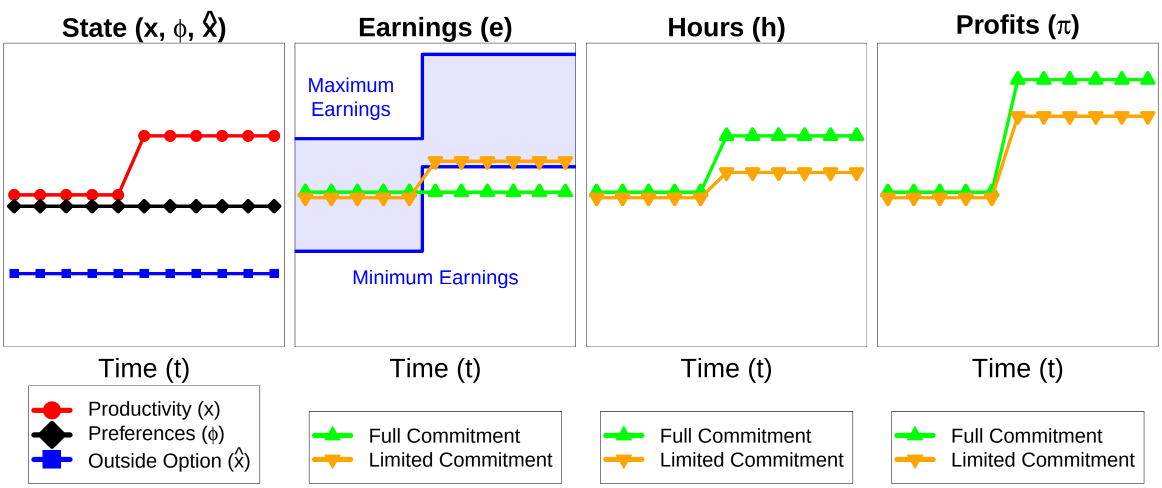
In the other extreme, suppose the employment contracts behave like spot market contracts where the worker chooses her own hours and was paid exactly what she produced. This would be the case if worker's outside option was always another firm with identical labor productivity as her current firm. In this case, the worker would set her hours such that the marginal benefit of additional consumption from additional production was equal to marginal cost of additional hours so that:

$$h_t = \left(\alpha \frac{\psi_i}{\theta_i} \frac{x_t^{1-\tau}}{\phi_t} \right)^{\frac{1}{1-\gamma+\alpha(1-\tau)}}, \qquad \text{and} \qquad e_t = \psi_i x_t h_t^{\alpha}. \tag{27}$$

Here, we see that the elasticity of hours with respect to shocks is smaller when workers set their own hours and consume their production compared to long-term contracts. Because hours variation will affect her consumption, the risk-averse worker attenuates the variability of her hours in response to shocks.

The variability of earnings and hours under long-term contracts constrained by limited commitment sits somewhere between these two extremes depending on the quality of the worker's outside option. If the worker has a better outside option, then it will be more likely that the firm and worker's commitment constraints will bind and require a change in earnings to maintain the match. Figure 2 shows how the contract behaves in response to an increase in the worker's productivity. If the contract was fully enforceable, then the contract increases the worker's hours without any increase in her earnings. Although the worker is worse off during the high productivity period because she receives no compensation for her additional hours, the firm benefits by taking the additional production as additional profits. The converse would happen in response to a negative productivity shock. Although either the firm or worker is worse of ex-post, they both benefit ex-ante from the risk-sharing agreement. However, if the worker had a better outside option ex-post, then she could threaten to separate from the firm because her hours are too high and earnings too low. Wanting to retain its worker, the firm increases the worker's earnings and reduces her hours to match her outside option value. Because the worker is risk-averse and has increasing marginal disutility of hours, the firm adjusts her earnings and hours not only during the period with high productivity but also in future periods to smooth these changes over time. These future promises allow the firm to optimally take advantage of the variation in productivity while providing value to the worker.

Figure 2: Impulse Response Functions



Note: This figure shows how earnings, hours, and profits respond in the model to an increase in labor productivity under full and limited commitment.

5 Estimation

In order to understand the sources of hours variation and create an environment for counterfactual analysis, this section estimates the model to match the dynamics of earnings, hours, and job mobility in the Survey of Income and Program Participation.

5.1 Externally Set Parameters

A period of time t is assumed to be four months as in the SIPP. I set the worker's relative risk-aversion in consumption $\tau \to 1$, so the worker has logarithmic utility in consumption. This is on the lower end of estimates in the literature (Layard et al. [2008]). However, because I do not allow workers to save, the desire for workers to smooth earnings across periods as measured by τ should be lower than in if the worker was able to save. I set the inverse of the Frisch elasticity $\gamma = 4$ based on the meta-analysis conducted by Elminejad et al. [2023]. I set the discount rate $\beta = 0.99$ for the four-month period, which implies an annual interest rate of 3.1%.

Table 6: Pre-Set Parameters

	Value
Discrete Period Length	Four Months
Coefficient of Relative Risk-Aversion (τ)	$\to 1$
Inverse Frisch Elasticity (γ)	4.0
Discount Factor (β)	0.99

Note: This table shows the values of the parameters set outside of estimation as described in section 5.1.

5.2 Parameterization and Identification

The model allows for permanent differences in individuals' productivity ψ_i and preferences for leisure θ_i that are jointly distributed according to $F_{\psi\theta}$. Because the model must be solved separately for each worker type i, I assume there are only nine types of workers in the economy for computational reasons. The distribution $F_{\psi\theta}$ is assumed to be a nine-point Gauss-Hermite approximation of a joint log-normal distribution.

$$\begin{bmatrix} \log(\psi_i) \\ \log(\theta_i) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\psi \\ \mu_\theta \end{bmatrix}, \begin{bmatrix} \sigma_\psi^2 & \rho_{\psi\theta}\sigma_\psi\sigma_\theta \\ \rho_{\psi\theta}\sigma_\psi\sigma_\theta & \sigma_\theta^2 \end{bmatrix}\right) \tag{28}$$

This yields five parameters to be estimated that underlie the distribution of worker types $(\mu_\psi, \mu_\theta, \sigma_\psi, \sigma_\theta, \rho_{\psi\theta})$. Because I use only nine-points, this will not be a good approximation of the joint normal distribution. However, it does allow for persistent differences in worker types to influence the distribution of earnings and hours. In order to estimate these parameters, I target the first and second order moments for the cross-section distribution of earnings and hours.

The Markov processes for preferences f_ϕ and match-specific labor productivity f_x are assumed to follow a logarithmic autoregressive process of order 1 with normally distributed stochastic term:

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t} \quad \epsilon_{x,t} \sim \mathcal{N}(0, \sigma_x^2), \tag{29}$$

$$\phi_{t+1} = \rho_\phi \phi_t + \epsilon_{\phi,t} \quad \epsilon_{\phi,t} \sim \mathcal{N}(0, \sigma_\phi^2). \tag{30}$$

Computationally, I approximate these distributions with a discrete Markov process with five states using the method proposed by Rouwenhorst [1995]. Relative to other methods, Rouwenhorst's method is more accurate in approximating an autoregressive process even when the persistence parameters are relatively high (Kopecky and Suen [2010]). In order to estimate the parameters $\{\rho_x, \sigma_x, \rho_\phi, \sigma_\phi\}$, I target the variability and persistence of changes in earnings and hours.

As shown in Proposition 1, when either productivity or preferences change, the optimal long-term contract will adjust the worker's hours in response. Thus, separately identifying these parameters is challenging. However, I argue that productivity shocks should have a larger impact on the variability of worker's earnings than preference shocks. This is because a preference shock impacts output only via a change in hours, while a productivity shock will impact output both through changes in hours and through the change in productivity. Because productivity shocks have a larger effect on output, it should be more difficult for firms to provide insurance against them as a result of limited commitment. Thus, productivity shocks should lead to more earnings volatility. Additionally, workers can escape bad match-specific labor productivity shocks by switching firms or switching to non-employment, so the match-specific labor productivity shock only affects the value of the current match. However, a leisure preference shock follows the worker into non-employment and to other firms, so it will impact both the current match and the worker's outside option values in similar ways. This allows for more the contract to provide more insurance against preference shocks compared to productivity shocks, so productivity shocks should cause less variability of earnings compared to productivity shocks.

The probabilities that workers contact firms when non-employed or employed in the model are given by λ_n and λ_e, respectively. In order to estimate these parameters, I target the fraction of employed workers who switch to a new firm (EE-rate) and the fraction of non-employed workers who become employed in the following period (NE-rate). In order to match the fraction of employed workers who become non-employed (NE-rate), I additionally include a probability δ that match-specific labor productivity enters an absorbing state of zero. When this happens, the firm and worker will certainly separate because the match will never be productive again. However, workers may transition to non-employment because of a high leisure preference shock or low labor productivity shock as well.

When workers contact a new firm, they draw an initial match-specific labor productivity $\hat{x}$ from a distribution F_x. Computationally, the match-specific labor productivity takes only finite values from the Rouwenhorst approximation with $N_x = 5$ points. I assume that the index of initial labor productivity is drawn from a truncated geometric distribution with probability p_x:

$$\mathbf{P}(\text{index}(\hat{x}) = j) = \frac{p_x(1-p_x)^j}{1-(1-p_x)^{N_x}}. \tag{31}$$

The probability p_x determines the leftward shift in the distribution of initial labor productivity. If $p_x = 1$, then the worker will certainly start at the lowest level of labor productivity. As p_x decreases, then the worker is more likely to start at a higher level of labor productivity. If $p_x = 0$, then the distribution of initial labor productivity is evenly distributed across the N_x productivity states. In order to estimate this parameter, I target average difference of earnings for workers who enter into non-employment next period compared to workers who just exited non-employment. For higher values of p_x, workers will have more match productivity growth over an employment spell, and thus have more earnings growth over a typical employment spell.

The final two parameters in the contracting model are the value of home production b and the returns to hours in production α. Home production b influences the value of non-employment, and thus has a large impact the earnings that workers are able to bargain for when they exit non-employment. If b is low, then workers will exit non-employment with low earnings. If b is higher, workers will exit non-employment with higher earnings. Thus, I target the difference between the average earnings of workers who just exited non-employment compared to average earnings of all workers. Returns to hours in production α measures how much changes in hours affect changes in output, and should be closely related to the elasticity of earnings with respect to hours. Thus, I target the covariance of changes in earnings and hours empirically.

In addition to the parameters in the structural model, I also allow for earnings and hours reported in the SIPP to contain measurement error ξ that is classical in logs with respect to true earnings and hours:

$$\log(\text{earnings}_{it}^{\text{report}}) = \log(\text{earnings}_{it}^{\text{true}}) + \xi_{iet}, \tag{32}$$

$$\log(\text{hours}_{it}^{\text{report}}) = \log(\text{hours}_{it}^{\text{true}}) + \xi_{iht}. \tag{33}$$

The measurement errors (ξ_{et}, ξ_{ht}) each have mean zero, variances of $(\sigma^2_{\xi e}, \sigma^2_{\xi h})$, and are uncorrelated with the true values of earnings and hours. Additionally, they are serially uncorrelated across time and with each other. In order to estimate these parameters, I target the auto-covariance of earnings and hours across four periods. As I show in appendix C.1, it is necessary to target the auto-covariance over four periods to identify a permanent, persistent, and transitory component of an observed stochastic process. Additionally, I target the covariance of the change in earnings experienced over four periods t to $t+3$ with the change in hours experienced over the inner two periods $t+1$ to $t+2$. As explained in section 3.1, this covariance should not be impacted by transitory changes such as measurement error, so should help disentangle transitory measurement error from true changes in earnings and hours.

5.3 Estimation Procedure and Results

There are 17 parameters χ to be estimated:

$$\chi = \{\mu_\psi, \sigma_\psi, \mu_\theta, \sigma_\theta, \rho_{\psi\theta}, \quad \rho_x, \sigma_x, \quad \rho_\phi, \sigma_\phi, \quad \lambda_n, \lambda_e, \delta, p_x \quad b, \quad \alpha \quad \sigma_{\xi e}, \sigma_{\xi h}, \}. \tag{34}$$

I estimate the model by simulated method of moments to select the parameters χ to minimize the distance between moments simulated by the model M_{sim} and corresponding moments estimated in the data M_{data} with a weight matrix W:

$$\chi^* = \arg\min_\chi (M_{\text{sim}}(\chi) - M_{\text{data}})' W (M_{\text{sim}}(\chi) - M_{\text{data}}). \tag{35}$$

I set the weight matrix whose diagonal terms are the inverse of the data moments and off-diagonal terms are zero. The optimization proceeds by differential evolution (Storn and Price [1997]) and results are shown in table 7.

Results of the estimation are shown in table 7 and the model fit is shown in table 8. The model is broadly able to match the first and second moments of the cross-sectional distribution of earnings and hours relatively well even with only nine individual types. While the model matches the EE- and EN-rates relatively well, it underestimates the NE-rate by 0.041 p.p. This leads the non-employment rate in the model economy to also be 6.3 percentage points higher than in the data. However, the estimated probability that non-employed workers are able to contact firms is near one. The reason why the model underestimates the NE-rate because the labor productivity that non-employed workers tend to draw are low enough that non-employment is preferable to the highest wage firms would be willing to pay. Although the NE-rate in the data and the estimated model is about 4.5%, the probability that match labor productivity enters an absorbing state of zero δ is only 2.8%. Thus,

Table 7: Estimation Results

	Parameter	Estimate
1.	Mean Productivity Type (μ_ψ)	1.880
2.	S.D. Productivity Type (σ_ψ)	0.860
3.	Mean Preference Type (μ_θ)	-17.989
4.	S.D. Preference Type (σ_θ)	1.469
5.	Corr. Productivity and Preference Type ($\rho_{\psi\theta}$)	0.589
6.	Auto-Corr. Productivity Shock (ρ_x)	0.237
7.	S.D. Productivity Shock (σ_x)	0.216
8.	Auto-Corr. Preference Shock (ρ_ϕ)	0.007
9.	S.D. Preference Shock (σ_ϕ)	0.432
10.	Non-employed Job Finding Rate (λ_n)	0.982
11.	Employed Job Finding Rate (λ_e)	0.250
12.	Exogenous Job Destruction Rate (δ_x)	0.028
13.	Distribution of Initial Labor Productivity (p_x)	0.317
14.	Non-employed home production ($\log b$)	5.260
15.	Production Returns to Hours (α)	0.983
16.	S.D. Meas. Error for Earnings ($\sigma_{\xi e}$)	0.272
17.	S.D. Meas. Error for Hours ($\sigma_{\xi h}$)	0.009

Note: This table shows the value of the parameters estimated by simulated method of moments as described in section 5.2.

about 40% of separation to non-employment happen endogenously as a result of the labor productivity shocks x of the leisure preference shocks ϕ.

The variability and persistence of changes in earnings and hours in the model and the data match relatively well. Interestingly, the estimate for the match-specific labor productivity shock process of 0.237 is low relative to most estimates in the literature. This is likely because of the high volatility of earnings observed in the SIPP from across the four-month survey waves that is highly correlated with hours. In order to rationalize this volatility, the model requires a volatile productivity shock process. Balke and Lamadon [2022] and Souchier [2022] estimate persistence of productivity to be greater than 0.8. However, they consider models at an annual frequency. As highlighted by the empirical work of Ganong et al. [2024], annual earnings are relatively stable despite high volatility in monthly earnings. Understanding the discrepancy between earnings volatility at annual sub-annual frequencies is an important area for future research.

Table 8: Model Fit

		Model	Data
1.	$\mathbf{E}[\log e_{it}]$	6.284	6.276
2.	$\mathbf{E}[\log h_{it}]$	3.626	3.665
3.	$\text{var}(\log e_{it})$	0.650	0.642
4.	$\text{var}(\log h_{it})$	0.089	0.100
5.	$\text{cov}(\log e_{it}, \log h_{it})$	0.118	0.125
6.	EE Rate	0.054	0.052
7.	EN Rate	0.048	0.045
8.	NE Rate	0.126	0.167
9.	$\text{var}(\Delta \log e_{it})$	0.191	0.198
10.	$\text{cov}(\Delta \log e_{it}, \Delta \log e_{it+1})$	-0.084	-0.074
11.	$\text{var}(\Delta \log h_{it})$	0.068	0.050
12.	$\text{cov}(\Delta \log h_{it}, \Delta \log h_{it+1})$	-0.030	-0.019
13.	$\text{cov}(\Delta \log e_{it}, \Delta \log h_{it})$	0.019	0.019
14.	$\text{cov}(\log e_{it}, \log e_{it+3})$	0.545	0.463
15.	$\text{cov}(\log h_{it}, \log h_{it+3})$	0.047	0.048
16.	$\text{cov}(\log e_{it+3} - \log e_{it}, \log h_{it+2} - \log h_{it+1})$	0.006	0.004
17.	$\mathbf{E}[\log e_{it}\|\text{NE}] - \mathbf{E}[\log e_{it}]$	-0.342	-0.609
18.	$\mathbf{E}[\log e_{it}\|\text{EN}] - \mathbf{E}[\log e_{it}\|\text{NE}]$	0.095	0.098

Note: This table shows the value of the parameters estimated by simulated method of moments as described in section 5.2. The variable h_{it} is the usual hours of work per week for individual i at time t, and e_{it} is weekly earnings in dollars adjusted to 2001 levels using the PCE.

The non-employed home production b is roughly 40% of average earnings for workers. This is consistent with estimates from previous literature looking at the change in earnings for workers entering non-employment in the SIPP and PSID (Rothstein and Valletta [2017], Braxton et al. [2024]). However, non-employed worker entering the workforce in the estimated model have earnings that are 0.342 lower compared to employed workers, which is about half the value in the data of 0.609. However, lower the value of b would reduce the covariance of changes in earnings and hours because participation constraints would be less likely to bind. The returns to hours in production α is estimated to be close to 0.983, which is much higher than previous estimates using data on relationship between earnings and hours (Yurdagul [2017], Bick et al. [2022], Shao et al. [2022]). In the model, the observed relationship between earnings and hours is less than the underlying relationship between output and hours as a result of insurance in the long-term employment contract. When shocks to productivity or preferences lead to an increase in hours, the optimal risk-sharing agreement will provide smooth earnings payments to the worker while, leading to a lower elasticity of earnings with respect to hours compared to the elasticity of output with respect to hours.

The estimates for the measurement error suggest that roughly 11.4% of variance of earnings is measurement error, while less than 1% of the variance of hours is measurement error. Empirically, there is much variability of earnings that is uncorrelated with changes in hours, and the model in the estimation partly interprets this as measurement error. When linking SIPP earnings records to tax records, Gottschalk and Huynh [2010] finds that the variance of measurement error to be 0.27% of the total variance of earnings. However, they also report that much of the measurement error in the SIPP is mean reverting, and the variance of earnings in the SIPP is in fact lower than the variance of earnings in tax records. Similarly, Bound et al. [2001] report that much of the measurement error in survey data is mean reverting. Because the model does not allow for mean reverting measurement error, it likely understates the true importance of measurement error. Future work should consider how incorporating a mean reverting component to measurement error impacts the results of the estimation.

6 Analysis

6.1 Pass-through of Productivity and Preference Shocks

The estimated model allows for an analysis of the extent to which changes in labor productivity and leisure preferences pass through to worker's hours, output, and earnings. Under perfect risk-sharing as discussed in section 4, worker's earnings should not respond to changes in productivity shocks x or leisure preference shocks ϕ, while hours are set as:

$$h_t = \left(e_t^{-\tau} \alpha \frac{\psi_i}{\theta_i} \frac{x_t}{\phi_t} \right)^{\frac{1}{1+\gamma-\alpha}} . \tag{36}$$

In the estimated model, the inverse Frisch elasticity γ is set to 4, and the returns to hours in production α is estimated to be 0.983. Thus, the elasticity of hours with respect to both productivity and preference shocks under full commitment would be 0.249. However, limited commitment will constrain the long-term contract, resulting in partial pass-through of shocks to earnings and reduced pass-through of shocks to hours.

To understand the actual pass-through of shocks in the estimated model, I run the following regression on the simulated data:

$$\log(z_{it}) = \beta_i + \beta_x \log(x_{it}) + \beta_\phi \log(\phi_{it}) + \epsilon_{it}. \tag{37}$$

Here, the outcome z_{it} is either hours, output, or earnings for worker i at time t, and β_i is a fixed effect for individual type. The estimated coefficients β_x and β_ϕ measure the elasticity of the outcome with respect to labor productivity. Results are shown in table 9.

Workers with a 1% higher preference shock have 0.243% lower hours, 0.239% lower output, and 0.023% lower earnings. This suggests that the insurance that workers receive from firms against changes in their preferences is quite close to the case of full commitment, where a 1% change in preferences would lead to a 0.249% change in hours, a 0.245% change in output, and no change in earnings. There are several reasons in the estimated model why firms are able to provide substantial insurance against preference shocks. First, preference shocks are worker specific and will follow worker to non-employment and other firms. This means that they impact both the value of the worker's current match and the value of the worker's outside option similarly. Thus, a preference shock is less likely to lead to a binding commitment constraint from the worker side. Additionally, the estimated persistence of preference shocks is very low at 0.007. This means that for any preference shock value experienced in the current periods, workers' preferences are likely to revert to their mean value in the next period, so a change in preferences today has limited impact on the total expected production of the worker going forward. Even if the worker works less today because of high leisure preferences, it is likely that they will be willing to work longer hours next period.

Compared to preference shocks, limited commitment has a larger impact on the pass-through of match-specific productivity shocks to outcomes. Under full commitment, workers with a 1% higher match-specific labor productivity shock should have 0.249% higher hours, 1.245% higher output, and no change in earnings. However, in the estimated model as shown in table 9, workers with a 1% higher productivity shock have 0.206% higher hours, 1.203% higher output, and 0.173% higher earnings. This suggests that the long-term contract under limited commitment is able to provide substantial insurance against productivity shocks, but not as much as against preference shocks. This is because productivity shocks do not affect the worker's outside option value. When match-specific productivity is high, the contract must increase the worker's earnings to respond to outside offers. Additionally, the estimated persistence of productivity shocks is 0.237, which is higher than the persistence of preference shocks. This means that a change in productivity today will have a larger impact on the total expected production of the worker going forward.

In addition to the regression coefficients shown in table 9, I also show how a positive shock to productivity and a negative shock to preferences pass through to outcomes over time in figures 3 and 4. Initially, the shocks lead to an increase in hours and output. However, worker's earnings remain relatively unchanged as a result of the risk-sharing in the long-term contract. Instead, the additional output is passed on to the firms' profits. Over time, however, the worker receives better outside offers from other firms in the economy, and their participation constraint binds. This leads to decreases in the workers' hours and output, but increases in workers' earnings.

Table 9: Pass-through of Shocks to Outcomes with Limited Commitment

	(1)	(2)	(3)
Productivity Shock $\log(x)$	0.206	1.203	0.173
Preference Shock $\log(\phi)$	-0.243	-0.239	-0.023
Outcome	log(hours)	log(output)	log(earnings)

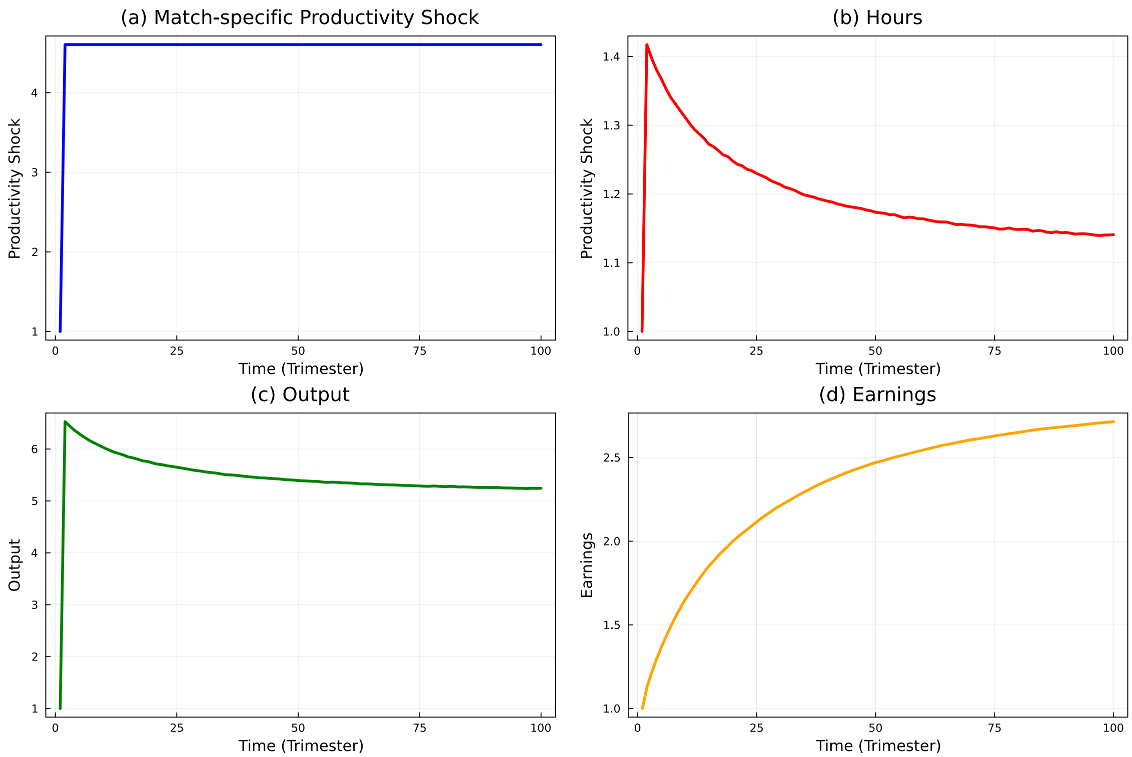
Note: This table show how outcomes differ for workers with different shocks to preferences and productivity in simulations of the estimated model by estimating regression coefficients for equation 37.

6.2 Variance Decomposition

The results of the previous section show how changes in shocks pass-through to workers outcomes, but they do not provide an understanding of the importance of these shocks for explaining different outcomes across workers. To understand the underlying sources of differences in earnings, hours, and output across workers, I perform a variance decomposition exercise in data simulated from the estimated model. To do this, I calculate for the regression in equation 37 the partial R-squared value for each explanatory variable: labor productivity shock x, leisure preference shock ϕ, and individual type i. The partial R-squared measures the fraction of the variance of the outcome that cannot be explained when that explanatory variable is removed from the regression. Results are shown in table 10.

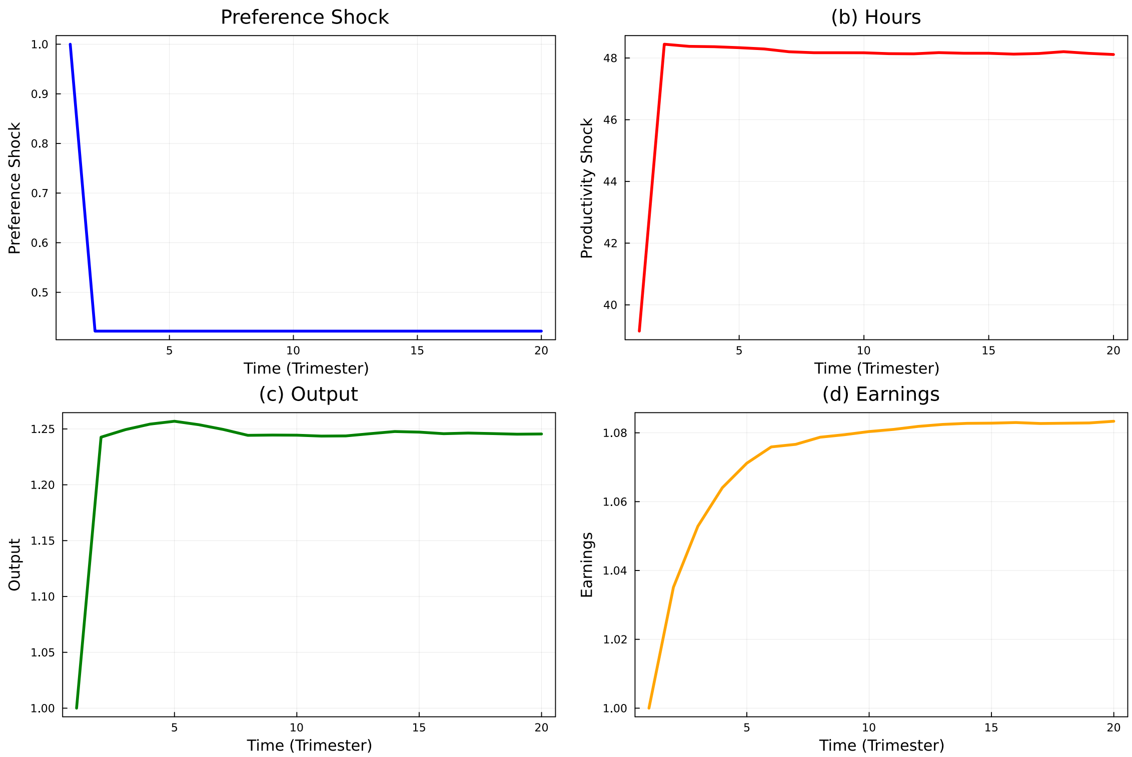
For all three outcomes, individual type i explains the largest portion of the variance. This is not surprising, because earnings and hours in the data are highly correlated across time for each individual. However, individual type explains substantially more of the variance in earnings compared to hours or output. For output, productivity shocks explain a sizeable

Figure 3: Average Response to Positive Productivity Shock



Note: This figure shows how hours, output, and earnings respond to a positive productivity shock in the estimated model. The match-specific productivity exogenously shifts from the median level of 1 to the highest level of 4.604. The path of hours, output, and earnings is shown for the next 100 periods. Transitions to non-employment are ruled out.

Figure 4: Average Response to Negative Preference Shock



Note: This figure shows how hours, output, and earnings respond to a negative leisure preference shock in the estimated model. The preference shock exogenously shifts from the median level of 1 to the lowest level of 0.422. The path of hours, output, and earnings is shown for the next 100 periods. Transitions to non-employment are ruled out.

portion of the variance. The difference in the variance decomposition for earnings and output arises in the model as a result of the risk-sharing in the long-term contract. Changes in labor productivity and leisure preferences result in changes in hours, but the firm insures the worker's earnings against the resulting changes in output.

Relative to leisure preference shocks, match-specific labor productivity shocks explain a larger fraction of the variance for all three outcomes. This is because the AR(1) process for match-specific labor productivity shocks is estimated to have a high variance of 0.216 and persistence of 0.237. This generates sizeable difference across in the match-specific labor productivity level.

Table 10: Partial R-Squared Variance Decomposition

	(1)	(2)	(3)
Individual Type i	0.448	0.869	0.394
Productivity Shock $\log x$	0.266	0.075	0.644
Preference Shock $\log \theta$	0.134	0.048	0.010
Outcome	log(hours)	log(earnings)	log(output)

Note: This table shows the partial R-squared for each outcome variable and explanatory variable in the estimated model. The partial R-squared for an explanatory variable measures the fraction of the variance for th outcome variable that cannot be explained by the other explanatory variables. The sum of partial R-squared values for a given outcome may sum to greater than 1 as a result of correlation between the explanatory variables. 5.2.

7 Conclusion

This paper develops a model of the labor market where earnings and hours are determined within a long-term contract motivated by risk-sharing. While previous work has provided reduced-form evidence that supports contracting (Beaudry and DiNardo [1995], Ham and Reilly [2002]), this paper estimates the model to match the dynamics of earnings, hours, and job mobility in the SIPP. Results of the estimation suggest that match-specific labor productivity shocks, which represent changes in firm's business needs, are the primary driver of changes in hours for workers. Firms also provide ample insurance to workers' earnings against changes in output because of these shocks.

The model also provides a useful starting point for considering why earnings instability differs across workers and the impact of government policies that restrict how workers' hours are set and adjusted in tandem will affect the labor market. Overtime penalties and fair workweek laws attempt to restrict firms from increasing their workers' hours in periods of high demand. In the model presented in this paper, this will lead to a reduction in firms' profits during these periods, and this would lead to a reduction in workers' earnings in all periods during contract negotiations. Mandating that firms provide paid leave to their workers would also lead to reductions in earnings as well as increases in hours in periods where leave is not taken.

The analysis provides several avenues for future research. First, the model assumes that firms are risk-neutral and can perfectly insure themselves against any profit fluctuations, small and young firms face imperfect financial markets that would make them less willing to provide smooth earnings to their workers. Future work can consider how the optimal long-term contract would behave when firms and workers are risk-averse. Second, the outside option for firms in the model is to terminate each worker and receive zero profits forever. However, hiring another worker to replace a current worker is real possibility that firms consider. Allowing for this would increase the frequency at which the firm's commitment constraints bind and reduce the insurance that they provide to workers. Finally, estimating the model on different subgroups of workers such as women and the high school educated could provide insights to whether the sources of hours variation and the extent to which firms insure earnings differs across workers.

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Table 11: Contract Types in the PSID

	Share of Answers
Salaried	0.400
Salary and Commissions	0.016
Paid by the Hour	0.523
Hourly and Tips	0.009
Hourly and Commissions	0.004
Other	0.047
Observations	5,269

Note: This table shows response to questions regarding payment scheme in the 2019 PSID for the main job heads of households and their spouses who are not self-employed. Observations are weighted using individual cross-section weights.

A Data Appendix

A.1 How are non-hourly workers paid?

The SIPP unfortunately only reports whether workers are paid by the hour or not. Non-hourly workers do not report the type of contract that they receive from their employer. However, a similar question in the Panel Survey of Income Dynamics (PSID) gives workers the option to choose fixed salary, hourly, commissions, and tips. I restrict the PSID sample to heads of households and their spouses in 2019 who are not self-employed. Additionally, I include only heads of households who are linked to an original PSID family. Table 11 shows that hourly and salary contracts alone account for 92.3% of all employment contracts. For the set of workers who choose that they are not paid by the hour, 86.3% report that they are paid a salary alone.

A.2 SIPP Subjective Question about Hours Changes

To understand the factors generating this variation in work hours within a job, I use two sets of subjective questions in the SIPP shown in Tables 12 and 13. First, workers in the SIPP are asked if they worked part-time (1-34 hours) in any weeks (Sunday to Saturday) during the survey wave. Those who responded affirmatively are further asked to choose a reason for working part-time from a menu of options. For workers who usually worked full-time, this is a reduction in their hours of work. Second, the SIPP asks workers if they were absent from their jobs without pay for any weeks (Sunday to Saturday) during the survey wave, and if yes, why. For all workers regardless of their usual hours, this represents a reduction in their working time. Together, the answers to reasons for part-time work and unpaid absences reveal that both demand-side factors (layoffs, business conditions, and slack work) and supply-side factors (illness and family care) are important sources of variations in hours within a job.

Table 12: Why worked less than 35 hours in a week?

	Share of Answers
Worked any part-time weeks this wave?	
Yes	0.117
If yes, why?	
Could not find full-time job	0.019
Wanted to work part time	0.022
Injury	0.018
Illness	0.090
Chronic Health Condition	0.010
Taking Care of Family	0.035
Full-time work week less than 35 hours	0.026
Slack work or material shortage	0.126
Job sharing arrangement	0.001
On vacation	0.471
In school	0.009
Other	0.173
Sample	Usually Full-time Hours
Observations	486,160

Note:
This table shows hours variation is caused by both demand and supply-side factors. The sample includes individuals usually working full-time, and their answers to if and why they worked part-time for at least one week during the survey wave. Observations are weighted using SIPP weights.

Table 13: Why absent from work without pay?

	Share of Answers
Any full week absences without pay this wave?	
Yes	0.046
If yes, why?	
On layoff	0.109
Slack work or business conditions	0.196
Own injury	0.030
Own illness	0.107
Pregnancy/childbirth	0.034
Taking care of children	0.021
On vacation/personal days	0.272
Bad weather	0.024
Labor dispute	0.004
New job to begin within 30 days	0.028
Job-sharing arrangement	0.002
Other	0.175
Sample	All workers
Observations	575,280

Note:
This table shows hours variation is caused by both demand and supply-side factors. The sample includes individuals with a job during the survey wave, and their answers to if and why they were absent from their job without pay. Observations are weighted using SIPP weights.

A.3 Changes in Hours by Subgroups

Table 14: Changes in Hours within a Job by Sex

	Mean $\lvert\Delta h\rvert$	Mean $\lvert\Delta \log(h)\rvert$	% $\lvert\Delta h\rvert \geq 5$	% $\lvert\Delta h\rvert \geq 10$	Observations
Female	2.48	0.08	23.1	10.0	375,338
Male	3.12	0.07	29.2	14.6	388,944

Note:
Sample includes individuals employed at same firm in two consecutive surveys. Observations are weighted using SIPP weights.

Table 15: Changes in Hours within a Job by Education Group

	Mean $\lvert\Delta h\rvert$	Mean $\lvert\Delta \log(h)\rvert$	% $\lvert\Delta h\rvert \geq 5$	% $\lvert\Delta h\rvert \geq 10$	Observations
1. No High School	3.03	0.09	26.2	13.3	62,636
2. High School	2.59	0.07	23.4	10.8	233,176
3. Some College	2.61	0.07	24.1	11.1	227,691
4. College	2.98	0.08	29.4	13.8	158,239
5. College Plus	3.59	0.09	35.1	17.6	82,540

Note:
Sample includes individuals employed at same firm in two consecutive surveys. Observations are weighted using SIPP weights.

Table 16: Changes in Hours within a Job by Age

	Mean $\lvert\Delta h\rvert$	Mean $\lvert\Delta \log(h)\rvert$	% $\lvert\Delta h\rvert \geq 5$	% $\lvert\Delta h\rvert \geq 10$	Observations
Age 25-29	2.99	0.09	28.0	13.0	93,668
Age 30-34	2.96	0.08	27.6	13.2	116,874
Age 35-39	2.88	0.08	26.9	12.7	128,378
Age 40-44	2.80	0.08	26.1	12.4	129,333
Age 45-49	2.74	0.07	25.6	12.0	118,428
Age 50-54	2.68	0.07	25.1	11.8	101,521

Note:
Sample includes individuals employed at same firm in two consecutive surveys. Observations are weighted using SIPP weights.

Table 17: Changes in Hours within a Job by Survey Month

	Mean $\lvert\Delta h\rvert$	Mean $\lvert\Delta \log(h)\rvert$	% $\lvert\Delta h\rvert \geq 5$	% $\lvert\Delta h\rvert \geq 10$	Observations
1. January	2.81	0.08	26.3	12.4	68,617
2. February	2.85	0.08	26.5	12.4	68,215
3. March	2.87	0.08	26.6	12.6	69,949
4. April	2.89	0.08	26.8	12.8	69,635
5. May	2.82	0.08	26.4	12.4	64,333
6. June	2.81	0.08	26.3	12.3	66,550
7. July	2.79	0.08	25.9	12.4	64,833
8. August	2.81	0.08	26.2	12.4	65,891
9. September	2.78	0.08	26.0	12.1	58,566
10. October	2.82	0.08	26.5	12.4	58,914
11. November	2.80	0.08	26.1	12.5	54,806
12. December	2.73	0.07	25.9	11.9	53,973

Note:

Sample includes individuals employed at same firm in two consecutive surveys. Observations are weighted using SIPP weights.

Table 18: Changes in Hours within a Job by Survey Year

	Mean $\lvert\Delta h\rvert$	Mean $\lvert\Delta \log(h)\rvert$	% $\lvert\Delta h\rvert \geq 5$	% $\lvert\Delta h\rvert \geq 10$	Observations
1990	2.97	0.08	26.4	12.3	29,805
1991	2.82	0.07	25.5	12.0	49,725
1992	2.88	0.08	26.2	12.4	57,611
1993	2.96	0.08	26.8	12.6	61,528
1994	2.85	0.08	26.0	12.3	44,997
1995	2.73	0.07	25.4	11.6	17,830
1996	3.35	0.09	30.7	14.3	35,740
1997	3.07	0.09	28.1	13.4	43,392
1998	2.90	0.08	27.3	12.8	43,667
1999	2.79	0.08	26.4	12.5	36,423
2001	3.23	0.09	30.2	14.3	38,990
2002	2.91	0.08	26.8	13.0	38,797
2003	2.71	0.08	25.8	12.2	25,870
2004	2.92	0.08	27.8	13.1	18,778
2005	2.78	0.08	26.1	12.5	18,869
2006	2.57	0.07	24.7	11.5	18,925
2007	2.44	0.07	23.3	11.0	12,180
2008	3.19	0.09	30.1	14.1	16,701
2009	2.86	0.08	26.8	12.8	38,939
2010	2.58	0.07	23.9	11.6	37,856
2011	2.40	0.07	23.0	10.6	33,680
2012	2.33	0.07	22.3	10.6	30,027
2013	2.11	0.06	20.9	9.1	13,952

Note:
Sample includes individuals employed at same firm in two consecutive surveys.
Observations are weighted using SIPP weights.

Table 19: Changes in Hours within a Job by Industry

	Mean $\|\Delta h\|$	Mean $\|\Delta \log(h)\|$	% $\|\Delta h\| \geq 5$	% $\|\Delta h\| \geq 10$	Observations
Agriculture	5.45	0.13	42.3	26.2	6,242
Army	6.04	0.12	45.1	29.2	3,993
Construction	3.06	0.08	28.5	15.3	36,834
Education	2.71	0.08	24.4	11.5	177,820
Finance	2.40	0.06	24.2	10.8	54,538
Government	1.83	0.05	16.1	8.0	53,148
Information	2.63	0.07	26.8	12.2	36,406
Leisure	3.57	0.11	33.5	14.9	37,210
Manufacturing	2.58	0.06	25.4	11.2	132,735
Mining	5.06	0.10	36.1	22.1	4,633
Other	3.30	0.10	30.0	14.3	24,151
Professional	3.02	0.08	28.5	13.9	54,051
Trade	2.94	0.08	28.3	12.6	100,019
Transportation	3.26	0.08	28.0	14.6	42,502

Note:
Sample includes individuals employed at same firm in two consecutive surveys. Observations are weighted using SIPP weights.

Table 20: Changes in Hours within a Job by Occupation

	Mean $\|\Delta h\|$	Mean $\|\Delta \log(h)\|$	% $\|\Delta h\| \geq 5$	% $\|\Delta h\| \geq 10$	Observations
Admin. Support	1.82	0.06	16.8	6.8	112,096
Army	5.92	0.12	44.0	28.4	3,948
Construction	2.97	0.08	26.3	14.5	31,663
Farming	5.33	0.13	41.4	25.8	4,058
Installation	2.55	0.06	23.5	11.2	31,183
Management	3.28	0.08	33.0	15.6	120,859
Production	2.41	0.06	22.5	9.7	73,975
Professional	2.86	0.08	27.4	12.9	204,065
Sales	3.15	0.09	30.8	14.1	46,402
Service	2.93	0.10	25.5	11.8	90,442
Transportation	3.60	0.09	30.4	16.3	45,591

Note:
Sample includes individuals employed at same firm in two consecutive surveys. Observations are weighted using SIPP weights.

B Model Appendix

B.1 The Optimal Long-term Contract

Following Marcet and Marimon ((2019)), we can rewrite the contracting problem in equation 14 recursively with two additional state variables: Pareto weights on the worker and firm utility. I then show that we must only keep track of only the worker's earnings in the preceding period. Suppose that W^* is the initial value that the worker receives from the cooperative negotiation and consider how the firm would decide to optimally deliver this value to the worker in a long-term contract.[6] The firm's problem is:

$$\max_{\mathcal{C}} \ (1-d_t)\Big(\pi_0 + \mathbf{E}\big[\sum_{t=1}^{\infty}\beta^t(1-D_t)(1-d_t)\pi_t \bigm| \eta^0\big]\Big), \tag{38}$$

subject to

1. Budget constraint:

$$e_t + \pi_t = y(h_t|x_t). \tag{39}$$

2. Firm's commitment constraint:

$$(1-d_t)\Big(\pi_t + \mathbf{E}\big[\sum_{k=1}^{\infty}\beta^k(1-D_{t+k})(1-d_{t+k})\pi_{t+k} \bigm| \eta^t\big]\Big) \geq 0. \tag{40}$$

3. Worker's commitment constraint:

$$\begin{aligned}(1-d_t)\Bigg(u(e_t,h_t|\phi_t) + \mathbf{E}\Big[\sum_{k=1}^{\infty}\beta^k\{(1-D_{t+k})(1-d_{t+k})u(e_{t+k},h_{t+k}|\phi_{t+k})\\ + (1-D_{t+k})d_{t+k}\mathcal{B}_t^W\} \Bigm| \eta^t\Big]\Bigg) + d_t\mathcal{B}_t^W \geq d_t\hat{W}_t.\end{aligned} \tag{41}$$

[6] It would alternatively be equivalent to define J^* to be the value that the firm receives from the cooperative negotiation and consider how the worker would deliver that value to the firm.

4. Worker receives cooperative negotiation value:

$$(1-d_0)\bigg(u(e_0,h_0|\phi_0)+\mathbf{E}\Big[\sum_{t=1}^{\infty}\beta^t\big\{(1-D_t)(1-d_t)u(e_t,h_t|\phi_t) \\ +(1-D_t)d_t\mathcal{B}_t^W\big\}\;\Big|\;\eta^0\Big]\bigg)+d_0\mathcal{B}_0^W \geq W^*. \quad (42)$$

This problem has the same set of constraints as the cooperative negotiation problem, so the solution to this problem will also be feasible for the cooperative negotiation game. The solution to this problem also will provide the value of the cooperative bargaining game to the worker, so it must also provide the value of the cooperative bargaining game to the firm. To see this, note that the contract that solves the cooperative negotiation game is feasible for this problem, so the firm must at least be better off here than they were in the cooperatively negotiated contract. However, if the firm was strictly better off here, then the cooperatively negotiated contract would not have been optimal. Thus, the contract solving the above problem is equivalent to the contract solving the cooperative negotiation game.

Let $\kappa_t^F(\eta^t)$ and $\kappa_t^W(\eta^t)$ be the Lagrange multipliers on the firm's and worker's commitment constraints at history η^t, respectively. Let μ be the Lagrange multiplier on the constraint that the worker initially receives a value W^* from the contract. Incorporating the constraints and Lagrange multipliers into the objective function, the firm's problem becomes:

$$\max_{\mathcal{C}}\;(1-d_0)\Bigg(\kappa_0^F\pi_0+(\kappa_0^W+\mu)\big(u(c_0,h_0)-\mathcal{B}^W(\eta_0)\big) \\ +\mathbf{E}\Bigg[\sum_{t=1}^{\infty}\beta^t\Big\{(1-D_t)(1-d_t)\big(K_t^F\pi_t+K_t^W u(c_t,h_t)-\kappa_t^W\mathcal{B}^W(\eta_t)\big) \\ +(1-D_t)d_tK_{t-1}^W\mathcal{B}^W(\eta_t)\Big\}\;\Bigg|\;\eta^0\Bigg]\Bigg), \quad (43)$$

such that for all histories η^t where the match has not yet been dissolved ($D_t=0$), the budget constraint is satisfied:

$$c_t+\pi_t=y(z,x_t,\psi,h_t), \quad (44)$$

and the Pareto weights for the worker and firm update according to:

$$K_t^W = K_{t-1}^W + \kappa_t^W, \tag{45}$$

$$K_t^F = K_{t-1}^F + \kappa_t^F. \tag{46}$$

Here, K_t^F and K_t^W are the promised Pareto weights for the firm and worker in each period. Over time, the Pareto weights increase for a party whenever they have a binding commitment constraint. The term κ_t^P for party $P \in \{F, W\}$ determining this update is the Lagrange multiplier on the party's commitment constraint. If the constraint is not binding, then this will be zero and the Pareto weight will not update. When the constraint binds, this will be the minimum positive number that is needed to satisfy the constraint. The initial condition is that the worker must receive the value from the cooperative negotiation game. Therefore, the worker's Pareto weight in the first period will be equal to the Lagrange multiplier on this constraint ($K_0^W = \mu$).

With this recursive reformulation of the contracting problem in hand, we can characterize the optimal contract as a function of the promised Pareto weights. First, I show how to write the problem recursively with one additional state variable: the relative Pareto weight on the firm $K_t = K_t^F/(K_t^F + K_t^W)$. The firm's recursive problem with two additional state variables is:

$$V(\eta, K^F, K^W) = \max_{\mathcal{C}} \quad K_{new}^F \pi + K_{new}^W u(c, h) + \beta \mathbf{E}[V(\eta', K_{new}^F, K_{new}^W)|\eta] - \kappa^W \mathcal{B}^W(\eta) \tag{47}$$

subject to the same constraints as before. Note that we can define a new value function as

$$\tilde{V}(\eta, K) = V(\eta, K^F, V^W)/(K^F + K^W), \tag{48}$$

where $K = K^F/(K^F + K^W)$ is the relative Pareto weight on the firm. Dividing equation 47 by $K^F + K^W$ and substituting in 48, we arrive at:

$$\tilde{V}(\eta, K) = \max_{\mathcal{C}} \ (1 + \tilde{\kappa}^F + \tilde{\kappa}^W)(K'\pi + (1 - K')u(c, h) + \beta \mathbf{E}[V(\eta', K')|\eta]) - \tilde{\kappa}^W \mathcal{B}^W(\eta) \quad (49)$$

subject to the same constraints as before. Here, $\tilde{\kappa}^P = \kappa^P/(K^F + K^W + \kappa^F + \kappa^W)$ for $P \in \{F, W\}$, and $K' = (K + \kappa^F)/(1 + \kappa^F + \kappa^W)$.

From the first order conditions of this problem, we can characterize the optimal decisions in the contract:

$$e = \left(\frac{K'}{1 - K'}\right)^{-\frac{1}{\tau}}, \quad (50)$$

and

$$h = \left(\alpha \frac{\psi_i}{\theta_i} \frac{x}{\phi} \frac{K'}{1 - K'}\right)^{\frac{1}{1-\alpha+\gamma}} \quad (51)$$

Here, we see that decisions depend on both the promised relative Pareto weight and the shocks to productivity in the match. Consumption depends only on the relative Pareto weight. When neither firm nor worker commitment constraint bind, consumption is constant within a match. Consumption will increase in response to a worker's binding commitment constraint, while the opposite is true when the firm's constraint binds. Hours depend both on the Pareto weight and the productivity shock. A perfect correlation exists between hours and productivity when neither constraint binds. Binding commitment constraints for the worker decrease hours, and binding commitment constraints for firms increase hours. The optimal contract decision rules highlight why productivity shocks and two-sided limited commitment can generate hours variation in the data that is not proportional to earnings variation.

From here, we can prove Proposition 1. Equation 50 shows that there is a one-to-one mapping between the relative Pareto weights K' and the worker's earnings e. For each state $\eta = \{\phi, x, \hat{x}\}$, the worker has an outside option value of $\hat{W}$. There exists a relative Pareto weight K at which the worker's earnings and hours are set such that she is indifferent between staying in the contract and going to her outside option. This maps to the minimum earnings $e_{\min}(\eta)$ that the worker must be paid. Similarly, the firm's outside option value in each period is to receive total expected profits of zero. There exists a relative Pareto weight K at which point the firm receives total expected value from staying in the contract of zero. This maps to the maximum earnings that the contract is willing to pay the worker.

Because the outside option for the worker $\hat{x}$ is observed before decisions are made, the contract will only dissolve if there is no Pareto weight K where the firm and worker's commitment constraints are satisfied simultaneously. This will be the case when the relative Pareto weight K that makes the worker indifferent between staying in the match is smaller than the relative Pareto weight that makes the firm indifferent between staying in the match. This would imply that the minimum earnings that the worker needs to receive is larger than the maximum earnings that firm is willing to pay. In cases where the match does not dissolve, the relative Pareto weight will adjust by as little as possible in order to satisfy the binding commitment constraint. In the case where the worker's commitment constraint binds, this means that the worker will receive the minimum earnings level. In cases where the firm's commitment constraint binds, this means that the worker will receive the maximum earnings level. When commitment constraints do not bind for either party, then the relative Pareto weight K will not update, so the earnings for the worker today will be the same as earnings for the worker in the previous period. When the match does not dissolve, hours are also set according to equation 51.

For the case where there is full commitment in Proposition 2, decisions in the optimal contract can be solved for similarly except there is no commitment constraints 40 and 41. The contract problem is only constrained by the budget constraint in equations 39 and that the worker receives the cooperatively negotiated value in equation 42. Thus, the Lagrange multipliers κ_t^F and κ_t^W will be zero in every period. The relative Pareto weight throughout all periods of the match where the contract does not dissolve would be $K = 1/(1+\mu)$, where μ is the Lagrange multiplier that the worker receives the cooperatively negotiated value in equation 42. Thus, earnings will be constant through the match from equation 50 and hours will be set optimally as in 51.

C Estimation Appendix

C.1 Identification in a Simpler Model

In order to provide some intuition for the identification of the contracting model, I derive point identification in a model where workers set their own hours and are paid according to what they produce. Here, it is possible to identify the processes governing the dynamics of earnings and hours using four periods of data.

Consider a large number of workers indexed by i whose earnings e_{it} and hours h_{it} are observed over four periods t, $t+1$, $t+2$, $t+3$. The data set is then $\{(e_{it}, h_{it})_{s=t}^{t+3}\}_{i=1}^{N_i}$. Workers' preferences for consumption and hours are given by:

$$u_i(c_{it}, h_{it}|\phi_{it}) = \log(c_{it}) - \theta_i \frac{h_{it}^{1+\gamma}}{1+\gamma}, \tag{52}$$

and their production is:

$$y_{it} = \psi_i x_{it} h_{it}^{\alpha}. \tag{53}$$

Here, θ_i and ψ_i are permanent shocks (types) for the workers' preferences and production that are jointly log-normal:

$$\begin{bmatrix} \log \theta_i \\ \log \psi_i \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\theta \\ \mu_\psi \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & \sigma_{\theta\psi}^2 \\ \sigma_{\theta\psi}^2 & \sigma_\psi^2 \end{bmatrix} \right). \tag{54}$$

The preference shock ϕ_{it} and productivity shock x_{it} processes follow an AR(1) process with a normally distributed innovation term:

$$\phi_{it+1} = \rho_\phi \phi_{it} + \epsilon_{it}^{\phi} \qquad \epsilon_{it}^{\phi} \sim \mathcal{N}(0, \sigma_\phi^2) \tag{55}$$

$$x_{it+1} = \rho_x x_{it} + \epsilon_{it}^{x} \qquad \epsilon_{it}^{x} \sim \mathcal{N}(0, \sigma_x^2) \tag{56}$$

These shocks are independent of each other and independent of the permanent types. Additionally, I assume that these shocks have reached a stationary distribution across workers, so their expectations and variances are:

$$\mathbf{E}[\phi_{it}] = 0, \tag{57}$$

$$\mathbf{E}[x_{it}] = 0, \tag{58}$$

$$\text{var}(\phi_{it}) = \frac{\sigma_\phi^2}{1-\rho_\phi^2}, \tag{59}$$

$$\text{var}(x_{it}) = \frac{\sigma_x^2}{1-\rho_x^2}, \tag{60}$$

Suppose workers set their own hours, are paid according to what they produce ($e_{it} = y_{it}$), and consume their earnings ($c_{it} = e_{it}$). The first order condition for the optimal choice of hours yields the following equation for hours:

$$\log(h_{it}) = \frac{1}{1+\gamma}\log\theta_i + \frac{1}{1+\gamma}\log\phi_{it} + \frac{1}{1+\gamma}\log\alpha. \tag{61}$$

The budget constraint yields the following equation for earnings:

$$\log(e_{it}) = \log\psi_i + \log x_{it} + \alpha\log h_{it}. \tag{62}$$

Moreover, earnings $\tilde{e}_{it}$ and hours $\tilde{h}_{it}$ as observed by the econometrician contain classical measurement error that is serially uncorrelated:

$$\log\tilde{e}_{it} = \log e_{it} + m_{it}^e \qquad m_{it}^e \sim \mathcal{N}(0, \sigma_{me}^2), \tag{63}$$

$$\log\tilde{h}_{it} = \log(h_{it}) + m_{it}^h \qquad m_{it}^h \sim \mathcal{N}(0, \sigma_{mh}^2). \tag{64}$$

I assume that γ is known, so there are twelve parameters to be estimated.

$$\{\mu_\psi, \mu_\theta, \sigma_\psi^2, \sigma_\theta^2, \sigma_{\psi\theta}^2, \rho_x, \sigma_x^2, \rho_\phi, \sigma_\phi^2, \sigma_{me}^2, \sigma_{mh}^2, \alpha\}. \tag{65}$$

These parameters can be estimated using a method of moment estimator derive from the four observations of earnings and hours.

First, consider the auto-covariance of hours over the four periods:

$$\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}) = \frac{1}{(1+\gamma)^2}\sigma_\theta^2 + \frac{\rho_\phi}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi} \tag{66}$$

$$\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) = \frac{1}{(1+\gamma)^2}\sigma_\theta^2 + \frac{\rho_\phi^2}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi} \tag{67}$$

$$\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) = \frac{1}{(1+\gamma)^2}\sigma_\theta^2 + \frac{\rho_\phi^3}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi} \tag{68}$$

Using these three equations, we can sequentially solve for $\rho_\phi, \sigma_\theta^2$, and σ_ϕ^2 as:

$$\rho_\phi = \frac{\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) - \mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2})}{\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1})}, \tag{69}$$

$$\sigma_\theta^2 = \frac{\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \rho_\phi \mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1})}{1-\rho_\phi}, \tag{70}$$

$$\sigma_\phi^2 = \frac{1-\rho_\phi^2}{\rho_\phi}\left(\mathrm{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}) - \sigma_\theta^2\right). \tag{71}$$

With these values in hand, we can use the equation for the variance of observed hours:

$$\mathrm{var}(\log \tilde{h}_{it}) = \frac{1}{(1+\gamma)^2}\sigma_\theta^2 + \frac{1}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi} + \sigma_{mh}^2 \tag{72}$$

to solve for the variance of the measurement error in hours as:

$$\sigma_{mh}^2 = \mathrm{var}(\log \tilde{h}_{it}) - \frac{1}{(1+\gamma)^2}\sigma_\theta^2 - \frac{1}{(1+\gamma)^2}\frac{\sigma_\phi^2}{1-\rho_\phi}. \tag{73}$$

Next, the covariance between changes in earnings and hours can be used to estimate the returns to hours in production.

$$\mathrm{cov}(\Delta \log \tilde{e}_{it}, \Delta \log \tilde{h}_{it}) = \alpha \mathrm{var}(\Delta \log h_{it}). \tag{74}$$

This implies:

$$\alpha = \frac{\text{cov}(\Delta \log \tilde{e}_{it}, \Delta \log \tilde{h}_{it})}{\text{var}(\Delta \log h_{it}) - 2\sigma^2_{mh}}. \tag{75}$$

Then, we can solve for the average leisure preference level μ_θ using the average hours in the economy:

$$\mathbf{E}[\log \tilde{h}_{it}] = \frac{1}{1+\gamma}(\mu_\theta + \log(\alpha)). \tag{76}$$

Before pinning down the random processes underlying labor productivity, we first need to pin down the covariance of the permanent leisure preference type θ_i and permanent labor productivity type ψ_i. This can be accomplished using the covariance of observed earnings and hours:

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{h}_{it}) = \frac{1}{1+\gamma}\sigma^2_{\psi\theta} + \alpha \text{var}(\log h_{it}), \tag{77}$$

This yields:

$$\sigma^2_{\psi\theta} = (1+\gamma)(\text{cov}(\log \tilde{e}_{it}, \log \tilde{h}_{it}) - \alpha \text{var}(\log h_{it})). \tag{78}$$

Now, consider the auto-covariance structure of earnings:

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) = \sigma^2_\psi + \rho_x \frac{\sigma^2_x}{1-\rho_x} + \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}) + \alpha\sigma^2_{\psi\theta} \tag{79}$$

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2}) = \sigma^2_\psi + \rho^2_x \frac{\sigma^2_x}{1-\rho_x} + \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) + \alpha\sigma^2_{\psi\theta} \tag{80}$$

$$\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+3}) = \sigma^2_\psi + \rho^3_x \frac{\sigma^2_x}{1-\rho_x} + \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) + \alpha\sigma^2_{\psi\theta} \tag{81}$$

Using these three equations, we can sequentially solve for ρ_x, σ^2_ψ, and σ^2_x as:

$$\rho_x = \frac{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+3}) - \text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2}) - \alpha^2(\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+3}) - \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}))}{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2}) - \text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) - \alpha^2(\text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+1}))}, \tag{82}$$

$$\sigma^2_\psi = \frac{\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+2}) - \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \alpha\sigma^2_{\psi\theta} - \rho_x(\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) - \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_i}{1-\rho_x} \tag{83}$$

$$\sigma_x^2 = \frac{1-\rho_x^2}{\rho_x}\left(\text{cov}(\log \tilde{e}_{it}, \log \tilde{e}_{it+1}) - \alpha^2 \text{cov}(\log \tilde{h}_{it}, \log \tilde{h}_{it+2}) - \alpha\sigma_{\psi\theta}^2 - \sigma_\psi^2\right). \tag{84}$$

With these values in had, we can use the equation for the variance of observed earnings:

$$\text{var}(\log \tilde{e}_{it}) = \sigma_\psi^2 + \frac{\sigma_x^2}{1-\rho_x} + \alpha^2(\text{var}(\log \tilde{h}_{it}) - \sigma_{mh}^2) + \alpha\sigma_{\psi\theta}^2 + \sigma_{me}^2 \tag{85}$$

to solve for the variance of the measurement error in earnings as:

$$\sigma_{me}^2 = \text{var}(\log \tilde{e}_{it}) - \sigma_\psi^2 - \frac{\sigma_x^2}{1-\rho_x} - \alpha^2(\text{var}(\log \tilde{h}_{it}) - \sigma_{mh}^2) - \alpha\sigma_{\psi\theta}^2. \tag{86}$$

Finally, the average labor productivity type μ_ψ can be solved from average earnings:

$$\mathbf{E}[\log \tilde{e}_{it}] = \mu_\psi + \alpha \log \tilde{h}_{it}. \tag{87}$$

In the model estimated in the paper, earnings and hours will further be affected by the risk-sharing between firms and workers within the long-term contract. However, by estimating the contract rates for meeting firms when non-employed λ_n and employed λ_e, we can hopefully infer how much risk-sharing firms and workers can do within the contract, and the back out the random processes underlying productivity and preferences.